Hong Kong Mathematics Olympiad (1990 – 91)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 求 $\log_3 14-\log_3 12+\log_3 486-\log_3 7$ 的值。
   Find the value of $\log_3 14-\log_3 12+\log_3 486-\log_3 7$.

2. 某科學家發現某樣本中細菌的數量每小時增加一倍。於下午四時，他發現細菌的數量為 3.2×10^8，若於同日正午該樣本中細菌的數量為 $N\times10^7$，求 N 的值。
   A scientist found that the population of a bacteria culture doubled every hour.
   At 4:00 pm , he found that the number of bacteria was 3.2×10^8 . If the number of bacteria in that culture at noon on the same day was $N\times10^7$, find the value of N .

3. 若 $x+\frac{1}{x}=8$，求 $x^3+\frac{1}{x^3}$ 的值。
   If $x+\frac{1}{x}=8$, find the value of $x^3+\frac{1}{x^3}$.

4. 若方程 $2x + 3y + a = 0$ 及 $bx - 2y + 1 = 0$ 代表同一直綫，求 $6(a + b)$ 的值。
   If the equations $2x + 3y + a = 0$ and $bx - 2y + 1 = 0$ represent the same line, find the value of $6(a + b)$.

5. 某童以每秒 2 米的速度由家步行回校，又以每秒 x 米的速度跑回家。
   若該童的往返平均速度為每秒 $2\frac{2}{3}$ 米，求 x 的值。
   A boy walks from home to school at a speed of 2 metres per second and runs back at x metres per second. His average speed for the whole journey is $2\frac{2}{3}$ metres per second.
   Find the value of x .

6. 直綫 $\frac{ax}{3}-\frac{2by}{5}=2a+b$ 恆過一定點 P，求 P 的 x 座標。
   The straight line $\frac{ax}{3}-\frac{2by}{5}=2a+b$ passes through a fixed point P. Find the x-coordinate of P.

7. 若一球體的直徑增加 20 %，則其體積增加 x %，求 x 的值。
   If the diameter of a sphere is increased by 20%, its volume will be increased by x %.
   Find the value of x .

8. 若 $\log_7[\log_5(\log_3 x)]=0$，求 x 的值。
   If $\log_7[\log_5(\log_3 x)]=0$, find the value of x .

9. 若 $\frac{7-8x}{(1-x)(2-x)}=\frac{A}{1-x}+\frac{B}{2-x}$，其中 x 為實數，且 $x\neq1$ 及 $x\neq2$，求 $A+B$ 的值。
   If $\frac{7-8x}{(1-x)(2-x)}=\frac{A}{1-x}+\frac{B}{2-x}$ for all real numbers x where $x \neq 1$ and $x \neq 2$,
   find the value of $A + B$.

10. 某商品的標價比成本高出 p%。在一次大減價中，店主以「照價八折」的價錢售出該商品。若該店主仍可得利潤 20%，求 p 的值。
    The marked price of an article is p% above its cost price. At a sale, the shopkeeper sells the article at 20% off the marked price. If he makes a profit of 20%, find the value of p .

11. 若 $a<0$，且 $2^{2a+4}-65\times2^a+4=0$，求 a 的值。
    If $a<0$ and $2^{2a+4}-65\times2^a+4=0$, find the value of a .

12. 設方程 $(x^2 - 11x - 10) + k(x + 2) = 0$ 的其中一根為零，求另一根。
    If one root of the equation $(x^2 - 11x - 10) + k(x + 2) = 0$ is zero, find the other root .

13. $[x]$ 是小於或等於 x 的最大整數。例如，$[6] = 6$，$[8.9] = 8$ 等。
若 $\left[\sqrt[4]{1}\right]+\left[\sqrt[4]{2}\right]+\cdots+\left[\sqrt[4]{n}\right]=n+2$，求 n 的值。
$[x]$ denotes the greatest integer less than or equal to x. For example, $[6] = 6$, $[8.9] = 8$, etc.
If $\left[\sqrt[4]{1}\right]+\left[\sqrt[4]{2}\right]+\cdots+\left[\sqrt[4]{n}\right]=n+2$, find the value of n.

14. a、b 為兩個不同之實數，且 $a^2 = 6a + 8$ 及 $b^2 = 6b + 8$，求 $\left(\frac{4}{a}\right)^2+\left(\frac{4}{b}\right)^2$ 的值。
a, b are two different real numbers such that $a^2 = 6a + 8$ and $b^2 = 6b + 8$.
Find the value of $\left(\frac{4}{a}\right)^2+\left(\frac{4}{b}\right)^2$.

15. $3^{12} - 1$ 可被一個大於 70 及小於 80 的整數所整除，求該整數。
$3^{12} - 1$ is divisible by an integer which is greater than 70 and smaller than 80. Find the integer.

16. 已知

$$2^3-1^3=3\times1^2+3\times1+1$$
$$3^3-2^3=3\times2^2+3\times2+1$$
$$4^3-3^3=3\times3^2+3\times3+1$$
$$\vdots \qquad \vdots$$
$$101^3-100^3=3\times100^2+3\times100+1$$

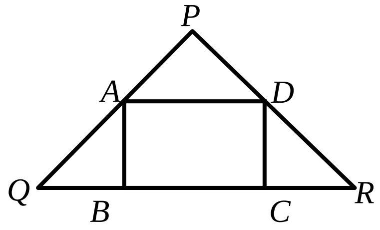
求 $1^2 + 2^2 + 3^2 + \cdots + 100^2$ 的值。

It is known that

$$2^3-1^3=3\times1^2+3\times1+1$$
$$3^3-2^3=3\times2^2+3\times2+1$$
$$4^3-3^3=3\times3^2+3\times3+1$$
$$\vdots \qquad \vdots$$
$$101^3-100^3=3\times100^2+3\times100+1$$

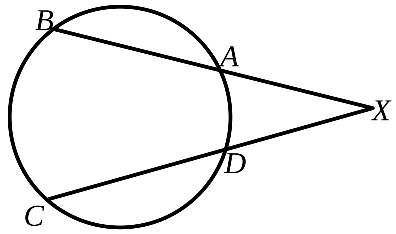
Find the value of $1^2 + 2^2 + 3^2 + \cdots + 100^2$.

17. 在圖一中，$PQ = PR = 8$ cm 及 $\angle QPR = 120°$。A、D 依次為 PQ、PR 的中點。若 $ABCD$ 是一個面積為 $\sqrt{x}$ cm^2 的矩形，求 x 的值。
In figure 1, $PQ = PR = 8$ cm and $\angle QPR = 120°$. A, D are the mid-points of PQ, PR respectively.
If $ABCD$ is a rectangle of area $\sqrt{x}$ cm^2, find x.



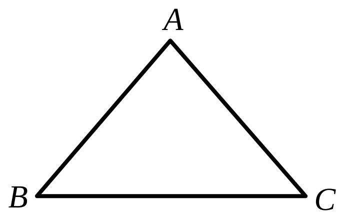
(Figure 1)(圖一)

18. 在圖二中，$XA = 10$ cm、$AB = 2$cm、$XD = 8$ cm 及 $DC = x$ cm，求 x 的值。
In figure 2, $XA = 10$ cm, $AB = 2$ cm, $XD = 8$ cm and $DC = x$ cm. Find the value of x.



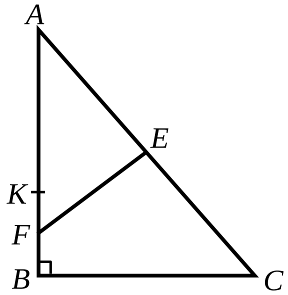
(Figure 2)(圖二)

19. 在圖三中，$AB = AC = 6$ cm 及 $BC = 9.6$ cm。若 ΔABC 的外接圓的直徑是 x cm，求 x 的值。
In figure 3, $AB = AC = 6$ cm and $BC = 9.6$ cm.
If the diameter of the circumcircle of ΔABC is x cm, find the value of x.



(Figure 3)(圖三)

20. 在圖四中，$\angle ABC = 90°$、$AK = BC$ 及 E、F 分別為 AC、KB 的中點。若 $\angle AFE = x°$，求 x 的值。
In figure 4, $\angle ABC = 90°$, $AK = BC$ and E, F are the mid-points of AC, KB respectively.
If $\angle AFE = x°$, find the value of x.



(Figure 4)(圖四)

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Hong Kong Mathematics Olympiad (1990 – 91)
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
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1. Find the units digit of 1357^{7890}.
求 1357^{7890} 的個位數。

2. If $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\cdots+\frac{1}{2450}=\frac{x}{100}$, find the value of x.
若 $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\cdots+\frac{1}{2450}=\frac{x}{100}$，求 x 的值。

3. $\frac{a}{3}$, $\frac{b}{4}$ and $\frac{c}{6}$ are three proper fractions in their simplest form, where a, b and c are positive integers. If c is added to the numerator of each fraction, then the sum of the fractions formed will be equal to 6. Find the value of $a+b+c$.
$\frac{a}{3}$、$\frac{b}{4}$ 及 $\frac{c}{6}$ 是三個化至最簡的真分數，其中 a 、b 及 c 是正整數。如果這三個分數的分子都加上 c，則所得三個分數的和是 6。求 $a+b+c$ 的值。

4. 細讀下列之帕斯卡三角形：

第 1 行	1
第 2 行	1 1
第 3 行	1 2 1
第 4 行	1 3 3 1
第 5 行	1 4 6 4 1
第 6 行	1 5 10 10 5 1
	⋮

求由第 1 行至第 15 行所有數的總和。

Study the Pascal's triangle shown below:

Row 1	1
Row 2	1 1
Row 3	1 2 1
Row 4	1 3 3 1
Row 5	1 4 6 4 1
Row 6	1 5 10 10 5 1
	⋮

Find the sum of all the numbers from Row 1 to Row 15.

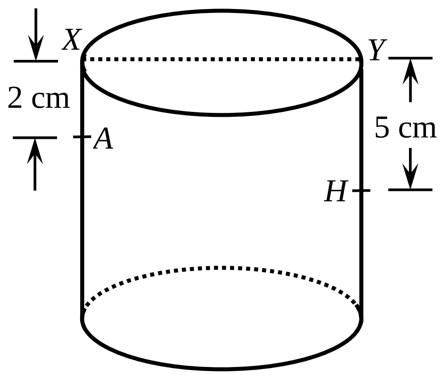
5. 在下列乘法算式中
□□□ × □□ = □□ × □□ = 5568，
每一方格代表由 1 至 9 的一個整數。若以上九個方格所代表的九個整數都不相同，求 □□□ 所代表的整數。
In the multiplication □□□ × □□ = □□ × □□ = 5568, each of the above boxes represents an integer from 1 to 9. If the integers for the nine boxes above are all different, find the number represented by □□□ .

6. 求 $1997^{1990}-1991$ 被 1996 除所得的餘數。
Find the remainder when $1997^{1990}-1991$ is divided by 1996 .

7. 求滿足不等式 $\sqrt{n}-\sqrt{n-1}<\frac{1}{80}$ 的 n 的最小正整數值。
Find the least positive integral value of n such that $\sqrt{n}-\sqrt{n-1}<\frac{1}{80}$.

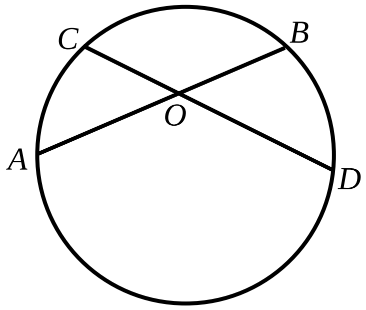
8. 方程 $32a+59b=3259$ 的其中一組正整數解 為 $(x, y)=(100, 1)$。現知僅有另一組正整數 (a, b) $(a\neq 100, b\neq 1)$ 使得 $32a+59b=3259$，求 a 的值。
One of the solutions of the equation $32a+59b=3259$ in positive integers is given by $(x, y)=(100, 1)$. It is known that there is exactly one more pair of positive integers (a, b) $(a\neq 100$ and $b\neq 1)$ such that $32a+59b=3259$. Find the value of a.

9. 在圖一中，XY 是圓柱形玻璃杯的直徑，杯底的圓周是 48 cm。杯外 A 點處 (在 X 之下 2 cm) 有一蟻，杯內 H 點處 (在 Y 之下 5 cm) 有一小滴蜜糖。若蟻行至蜜糖的最短路綫長 x cm，求 x。(杯的厚度可略去不計。)
In figure 1, XY is a diameter of a cylindrical glass, 48 cm in base circumference. On the outside is an ant at A, 2 cm below X and on the inside is a small drop of honey at H, 5 cm below Y. If the length of the shortest path for the ant to reach the drop of honey is x cm, find x. (Neglect the thickness of the glass.)



(Figure 1)(圖一)

10. 在圖二中，弦 AOB、COD 相交於 O。若過 A 的切綫與過 C 的切綫相交於 X，過 B 的切綫與過 D 的切綫相交於 Y，且$\angle AXC=130^\circ$、$\angle AOD=120^\circ$、$\angle BYD=k^\circ$，求 k 的值。
In figure 2, two chords AOB, COD cut at O. If the tangents at A and C meet at X, the tangents at B and D meet at Y and $\angle AXC=130^\circ$, $\angle AOD=120^\circ$, $\angle BYD=k^\circ$, find the value of k.



(Figure 2)(圖二)

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