Hong Kong Mathematics Olympiad (1992 – 93)
Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i)　　已知 $7^{2x} = 36$ 及 $7^{-x} = (6)^{\frac{-a}{2}}$，求 a 的值。

　　　Given that $7^{2x} = 36$ and $7^{-x} = (6)^{\frac{-a}{2}}$, find the value of a.

$a =$

(ii)　　若 $\log_2\{\log_2[\log_2(2b) + a] + a\} = a$，求 b 的值。
　　　Find the value of b if $\log_2\{\log_2[\log_2(2b) + a] + a\} = a$.

$b =$

(iii)　　若方程 $(x - b)(x - 2)(x + 1) = 3(x - b)(x + 1)$ 正根的總數為 c，求 c 的值。
　　　If c is the total number of positive roots of the equation
　　　$(x - b)(x - 2)(x + 1) = 3(x - b)(x + 1)$, find the value of c.

$c =$

(iv)　　若 $\sqrt{3 - 2\sqrt{2}} = \sqrt{c} - \sqrt{d}$，求 d 的值。
　　　If $\sqrt{3 - 2\sqrt{2}} = \sqrt{c} - \sqrt{d}$, find the value of d.

$d =$

FOR OFFICIAL USE

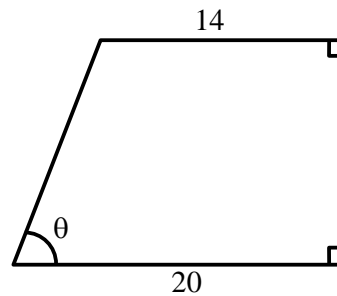| Score for accuracy		×	Mult. factor for speed		=		Team No.	
		+	Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1992 – 93)
Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\sin \theta = \dfrac{4}{5}$，求四邊形面積 a。

If $\sin \theta = \dfrac{4}{5}$, find a, the area of the quadrilateral.

14

20

θ

$a =$

(ii) 若 $b = 126^2 - a^2$，求 b 的值。
If $b = 126^2 - a^2$, find the value of b.

$b =$

(iii) 將$(3000 + b)$按 $5 : 6 : 8$ 分成 3 份，最小的一份為c。求 c 的值。
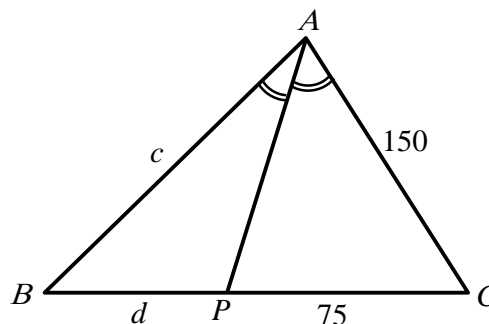Dividing $(3000 + b)$ in a ratio $5 : 6 : 8$, the smallest part is $ c$.
Find the value of c.

$c =$

(iv) 圖中 AP 等分$\angle BAC$。已知 $AB = c$，
$BP = d$，$PC = 75$ 及 $AC = 150$，求 d 的值。
In the figure, AP bisects $\angle BAC$.
Given that $AB = c$, $BP = d$, $PC = 75$ and
$AC = 150$, find the value of d.

A

c

150

B

d

P

75

C

$d =$

Hong Kong Mathematics Olympiad (1992 – 93)
Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 a 為以 13 除 2614303940317 的餘數，求 a 的值。
If a is the remainder when 2614303940317 is divided by 13, find the value of a .

$a =$

(ii) 設 $P(x, b)$ 為直綫 $x + y = 30$ 上的點且滿足 OP 斜率為 a (O 乃原點)。求 b 的值。
Let $P(x, b)$ be a point on the straight line $x + y = 30$ such that slope of $OP = a$ (O is the origin). Determine the value of b .

$b =$

(iii) 兩人踏單車，起始時相距 $(b + 26)$ km，以時速 40 km/h 及 60 km/h 相向而行。一蒼蠅以時速 100 km/h 往返兩人鼻尖，若牠在兩人碰上前共飛 c km，求 c 的值。
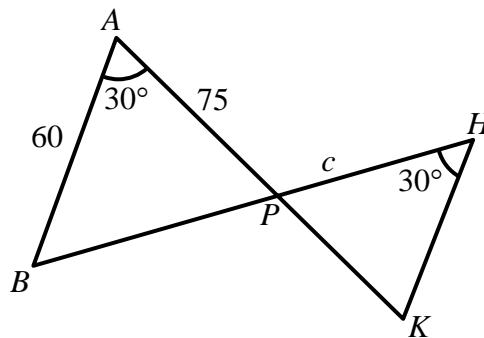Two cyclists, initially $(b + 26)$ km apart travelling towards each other with speeds 40 km/h and 60 km/h respectively. A fly flies back and forth between their noses at 100 km/h. If the fly flied c km before crushed between the cyclists, find the value of c .

$c =$

(iv) 圖中 APK 及 BPH 為直綫。若 $d = \Delta HPK$ 的面積，求 d 的值。
In the figure, APK and BPH are straight lines. If $d =$ area of triangle HPK, find the value of d.

$d =$

A

$30°$ 75

H

60

c

$30°$

P

B

K

Hong Kong Mathematics Olympiad (1992 – 93)
Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 x 和 y、y 和 z、z 和 x 的平均值分別為 5、9、10。
若 x、y、z 的平均值是 a，求 a 的值。
Given that the means of x and y, y and z, z and x are respectively 5, 9, 10.
If a is the mean of x, y, z, find the value of a .

$a =$

(ii) 某兩數的比例為 $5 : a$。當每邊加 12 時，兩數的比例變為 $3 : 4$。
若 b 為原本兩數之差及 $b > 0$，求 b 的值。
The ratio of two numbers is $5 : a$. If 12 is added to each of them, the ratio becomes $3 : 4$.
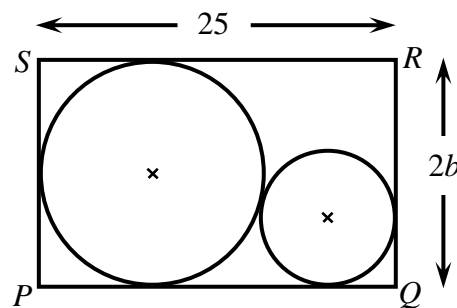If b is the difference of the original numbers and $b > 0$, find the value of b .

$b =$

(iii) $PQRS$ 為一長方形，
若細圓的半徑為 c，求 c 的值。
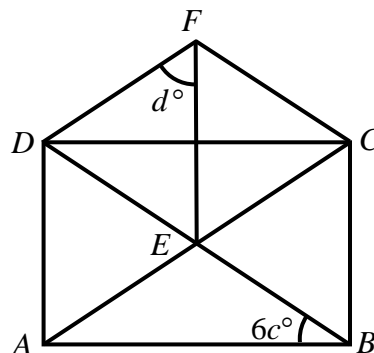$PQRS$ is a rectangle. If c is the radius of the smaller circle, find the value of c.

$c =$

(iv) $ABCD$ 為一長方形及 CEF 為一等邊三角形，
$\angle ABD = 6c°$，求 d 的值。
$ABCD$ is a rectangle and CEF is an equilateral triangle,
$\angle ABD = 6c°$, find the value of d .

$d =$

Hong Kong Mathematics Olympiad (1992 – 93)
Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 長方形兩對邊同時加長 50%，而其餘兩對邊則縮短 20%。
若長方形的面積增加 $a\%$，求 a 的值。
Two opposite sides of a rectangle are increased by 50% while the other two are decreased by 20%. If the area of the rectangle is increased by $a\%$, find the value of a.

$a =$

(ii) 設 $f(x) = x^3 - 20x^2 + x - a$ 及 $g(x) = x^4 + 3x^2 + 2$。若 $h(x)$ 為 $f(x)$ 和 $g(x)$ 的最大公因子，
求 $b = h(1)$ 的值。
Let $f(x) = x^3 - 20x^2 + x - a$ and $g(x) = x^4 + 3x^2 + 2$.
If $h(x)$ is the highest common factor of $f(x)$ and $g(x)$, find the value of $b = h(1)$.

$b =$

(iii) It is known that $b^{16} - 1$ has four distinct prime factors,
determine the largest one, denoted by c.
已知 $b^{16} - 1$ 共有四質因子，求其中最大的一個，以 c 表它。

$c =$

(iv) When c is represented in binary scale, there are d '0's. Find the value of d.
當以二進制表示 c，則其中有 d 個'0'。求 d 的值。

$d =$

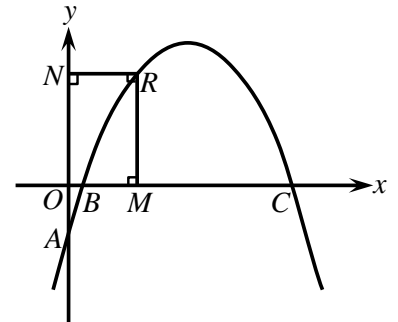Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

右圖所示為 $y = px^2 + 5x + p$ 的圖像。$A = (0, -2)$、$B = \left(\dfrac{1}{2}, 0\right)$、

$C = (2, 0)$、$O = (0, 0)$。

The following shows the graph of $y = px^2 + 5x + p$. $A = (0, -2)$,

$B = \left(\dfrac{1}{2}, 0\right)$, $C = (2, 0)$, $O = (0, 0)$.

(i)　求 p 的值。

　　Find the value of p .

$p =$

(ii)　若 y 的最大值為 $\dfrac{9}{m}$，求 m 的值。

　　If $\dfrac{9}{m}$ is the maximum value of y, find the value of m .

$m =$

(iii)　設 R 為曲線上一點且 $OMRN$ 為一正方形。若 R 的 x 坐標為 r，求 r 的值。

　　Let R be a point on the curve such that $OMRN$ is a square.

　　If r is the x-coordinate of R , find the value of r .

$r =$

(iv)　一斜率為 -2 及通過原點的直線與上述曲線相交於兩點 E 及 F。若 EF 中點的 y 坐標為 $\dfrac{7}{s}$，求 s 的值。

　　A straight line with slope $= -2$ passes through the origin cutting the curve at two points E and F.

　　If $\dfrac{7}{s}$ is the y-coordinate of the midpoint of EF, find the value of s .

$s =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
			Total score					Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

$OABC$ 為一四面體，其中 OA、OB 及 OC 互相垂直。
已知 $OA = OB = OC = 6x$。
$OABC$ is a tetrahedron with OA, OB and OC being mutually perpendicular.
Given that $OA = OB = OC = 6x$.
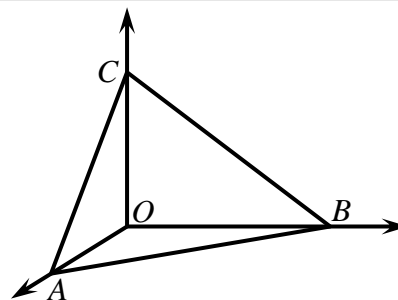
(i) 若 $OABC$ 的體積為 ax^3，求 a 的值。
If the volume of $OABC$ is ax^3, find the value of a.

$a =$

(ii) 若 $\triangle ABC$ 的面積為 $b\sqrt{3}x^2$，求 b 的值。
If the area of $\triangle ABC$ is $b\sqrt{3}x^2$, find the value of b.

$b =$

(iii) 若由 O 至 $\triangle ABC$ 的距離為 $c\sqrt{3}x$，求 c 的值。
If the distance from O to $\triangle ABC$ is $c\sqrt{3}x$, find the value of c.

$c =$

(iv) 若由 C 至 AB 中點的俯角為 θ，且 $\sin\theta = \dfrac{\sqrt{d}}{3}$，求 d 的值。

If θ is the angle of depression from C to the midpoint of AB and $\sin\theta = \dfrac{\sqrt{d}}{3}$,

find the value of d.

$d =$

Hong Kong Mathematics Olympiad (1992 – 93)
Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

已知方程 $x^2 + (m + 1)x - 2 = 0$ 有兩整數根 $(α + 1)$ 及 $(β + 1)$，且 $α < β$ 及 $m ≠ 0$。設 $d = β - α$。

Given that the equation $x^2 + (m + 1)x - 2 = 0$ has 2 integral roots $(α + 1)$ and $(β + 1)$ with $α < β$ and $m ≠ 0$.

Let $d = β - α$.

(i) 求 m 的值。
Find the value of m.

$m =$

(ii) 求 d 的值。
Find the value of d.

$d =$

設 n 為由 1 至 2000 內被 3 或 7 除時，餘數都為 1 的整數的總數。

Let n be the total number of integers between 1 and 2000 such that each of them gives a remainder of 1 when it is divided by 3 or 7.

(iii) 求 n 的值。
Find the value of n.

$n =$

(iv) 若 s 為上述 n 個整數的總和，求 s 的值。
If s is the sum of all these n integers, find the value of s.

$s =$

FOR OFFICIAL USE

Score for accuracy ☐ × Mult. factor for speed ☐ = ☐   Team No. ☐

+ Bonus score ☐   Time ☐ ☐

Total score ☐   Min.   Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。
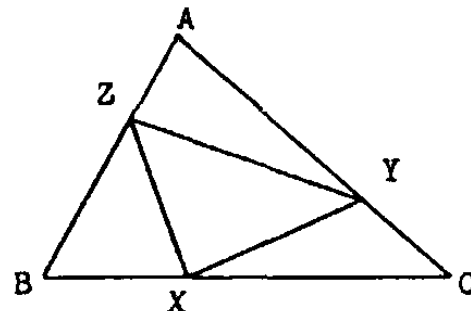
點 X、Y、Z 依次將 BC、CA、AB 分成 $1 : 2$。

設 ΔAZY 的面積 : ΔABC 的面積 $= 2 : a$ 及

ΔAZY 的面積 : ΔXYZ 的面積 $= 2 : b$。

BC, CA, AB are divided respectively by the points X, Y, Z in the ratio $1 : 2$.

Let area of ΔAZY : area of $\Delta ABC = 2 : a$ and

area of ΔAZY : area of $\Delta XYZ = 2 : b$.

(i) 求 a 的值。
Find the value of a .

$a =$

(ii) 求 b 的值。
Find the value of b .

$b =$

擲一枚骰子兩次。設 $\dfrac{x}{36}$ 為擲得點數總和為 7 或 8 的概率，$\dfrac{y}{36}$ 為擲得兩數之差為 1 的概率。

A die is thrown 2 times. Let $\dfrac{x}{36}$ be the probability that the sum of numbers obtained is 7 or 8 and $\dfrac{y}{36}$ be the probability that the difference of numbers obtained is 1.

(iii) 求 x 的值。
Find the value of x .

$x =$

(iv) 求 y 的值。
Find the value of y .

$y =$

FOR OFFICIAL USE

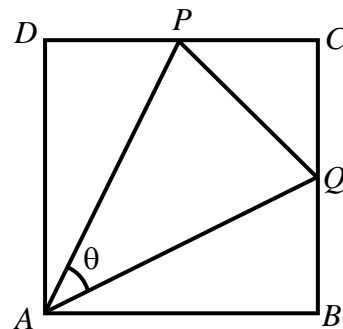| Score for accuracy		$\times$	Mult. factor for speed		$=$		Team No.	
			$+$	Bonus score			Time	
		Total score					Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

ABCD 乃一邊長為 $20\sqrt{5}x$ 的正方形。P、Q 分別為 *DC* 及 *BC* 的中點。
ABCD is a square of side length $20\sqrt{5}x$. *P*, *Q* are midpoints of *DC* and *BC* respectively.

(i) 若 $AP = ax$，求 a 的值。
If $AP = ax$, find the value of a.

$a =$

(ii) 若 $PQ = b\sqrt{10}x$，求 b 的值。
If $PQ = b\sqrt{10}x$, find the value of b.

$b =$

(iii) 若由 A 至 PQ 的距離為 $c\sqrt{10}x$，求 c 的值。
If the distance from *A* to *PQ* is $c\sqrt{10}x$, find the value of c.

$c =$

(iv) 若 $\sin\theta = \dfrac{d}{100}$，求 d 的值。
If $\sin\theta = \dfrac{d}{100}$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		

+ Bonus score

Total score

Min.   Sec.