Hong Kong Mathematics Olympiad (1994 – 95)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 求 1234567654321 的平方根。
Find the positive square root of 1234567654321.

2. 已知 $f\left(\frac{1}{x}\right)=\frac{x}{1-x^2}$，求 f(2) 的值。
Given that $f\left(\frac{1}{x}\right)=\frac{x}{1-x^2}$, find the value of f(2).

3. 解 $3^{2x}+9=10(3^x)$。
Solve $3^{2x}+9=10(3^x)$.

4. 隨意抽出一個三位數。求這個數是一完全平方數的機會。
A three-digit number is selected at random. Find the probability that the number selected is a perfect square.

5. 已知 $\sin x+\cos x=\frac{1}{5}$，且 $0\le x\le\pi$，求 $\tan x$ 的值。
Given that $\sin x+\cos x=\frac{1}{5}$ and $0\le x\le\pi$, find the value of $\tan x$.

6. 有多少對正整數 x、y 可滿足 $xy-3x-2y=10$？
How many pairs of positive integers x, y are three satisfying $xy-3x-2y=10$?

7. x、y 為正整數，且 $3x+5y=123$。求 $|x-y|$ 的最小值。
x, y are positive integers and $3x+5y=123$. Find the least value of $|x-y|$.

8. 求 1997^{913} 被 10 除時所得的餘數。
Find the remainder when 1997^{913} is divided by 10.

9. 如圖一，若 $BC=3DE$，求 r 的值，其中 $r=\frac{\text{Area of }\Delta ADE}{\text{Area of }\Delta BDC}$。
In figure 1, if $BC=3DE$, find the value of r,
where $r=\frac{\text{Area of }\Delta ADE}{\text{Area of }\Delta BCD}$.

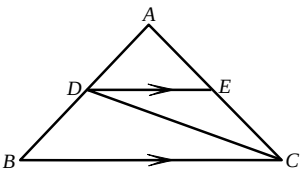

Figure 1 圖一

10. 如圖二，A、B、C、D 為直角三角形 PQR 各邊上的點。若 $ABCD$ 為一正方形，且 $QA=8$ 及 $BR=18$，求 AB 的值。
A, B, C, D are points on the sides of the right-angled triangle PQR as shown in figure 2.
If $ABCD$ is a square, $QA=8$ and $BR=18$, find the value of AB.

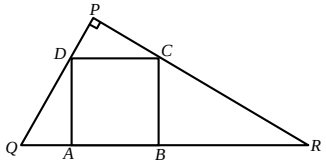

Figure 2 圖二

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Hong Kong Mathematics Olympiad (1994 – 95)
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
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每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 求方程 $x^3 + (x+1)^3 + (x+2)^3 = (x+3)^3$ 的正整數解數目。

   Find the number of positive integral solutions of the equation $x^3 + (x+1)^3 + (x+2)^3 = (x+3)^3$.

2. 如圖一，四邊形 $ABCD$ 的對角綫交於 O。
   若 $\angle AOB = 30°$、$AC = 24$ 及 $BD = 22$，
   求四邊形 $ABCD$ 的面積。
   In figure 1, $ABCD$ is a quadrilateral whose diagonals intersect at O. If $\angle AOB = 30°$, $AC = 24$ and $BD = 22$, find the area of the quadrilateral $ABCD$.

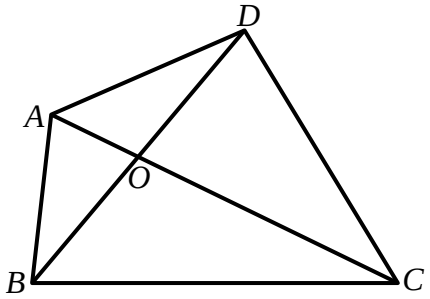

Figure 1 圖一

3. 已知 $\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\cdots+\frac{n-1}{n}=\frac{n-1}{2}$，求 $\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\cdots+\left(\frac{1}{10}+\cdots+\frac{9}{10}\right)$ 的值。

   Given that $\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\cdots+\frac{n-1}{n}=\frac{n-1}{2}$, find the value of

   $\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\cdots+\left(\frac{1}{10}+\cdots+\frac{9}{10}\right)$.

4. 若 x 及 y 為正整數，且 $x^2 = y^2 + 2000$，求 x 的最小值。

   Suppose x and y are positive integers such that $x^2 = y^2 + 2000$, find the least value of x .

5. 已知 37^{100} 為一 157 位數，且 37^{15} 為一 n 位數，求 n 的值。

   Given that 37^{100} is a 157-digit number, and 37^{15} is an n-digit number. Find the value of n .

6. 已知 $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6}(n+1)(2n+1)$，求 $19\times21 + 18\times22 + 17\times23 + \cdots + 1\times39$ 的值。

   Given that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n}{6}(n+1)(2n+1)$,

   find the value of $19\times21 + 18\times22 + 17\times23 + \cdots + 1\times39$.

7. 在圖中， $ABCD$ 為一正方形，且 $AB=1$ 及 CPQ 為一等邊三角形。求 ΔCPQ 的面積。
In figure 2, $ABCD$ is a square where $AB=1$ and CPQ is an equilateral triangle. Find the area of ΔCPQ .

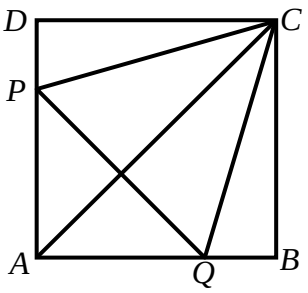

Figure 2 圖二

8. 用一元、二元及五元硬幣，以湊合十七元，且每次均須使用各種硬幣，其方法有 n 種，求 n 的值。
The number of ways to pay a sum of $17 by using $1 coins, $2 coins and $5 coins is n. Find the value of n . (Assume that all types of coins must be used each time.)

9. 圖三是一個 3×3 的正方形，求圖中三角形的總數。
In figure 3, find the total number of triangles in the 3×3 square.

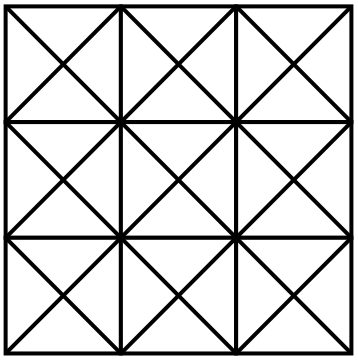
Figure 3 圖三

10. 在圖四中，象限的半徑和大半圓的直徑是 2，求小半圓的半徑。
In figure 4, the radius of the quadrant and the diameter of the large semi-circle is 2. Find the radius of the small semi-circle.

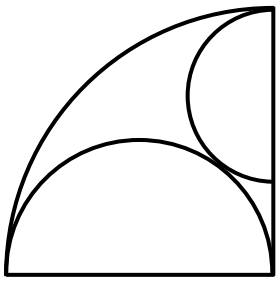
Figure 4 圖四

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