Hong Kong Mathematics Olympiad (1995-96)
Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

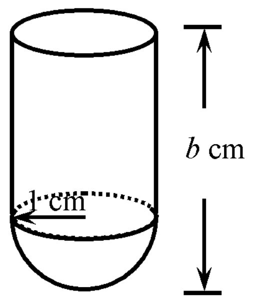
(i) 若一個等邊三角形與一個正六邊形周長相等，而其面積比為 $2 : a$，求 a 的值。
The perimeter of an equilateral triangle is exactly the same in length as the perimeter of a regular hexagon. The ratio of the areas of the triangle and the hexagon is $2 : a$, find the value of a.

a =

(ii) 若 $5^x + 5^{-x} = a$ 和 $5^{3x} + 5^{-3x} = b$ 求 b 的值。
If $5^x + 5^{-x} = a$ and $5^{3x} + 5^{-3x} = b$, find the value of b.

b =

(iii) 圖中為一圓柱體和半球體組成的無蓋空心物體。半球體和圓柱體的半徑均為 1 cm。若這物體的長度為 b cm，且表面面積為 $c\pi$ cm^2，求 c 的值。
The figure shows an open cylindrical tube (radius = 1 cm) with a hemispherical bottom of radius 1 cm. The height of the tube is b cm and the external surface area of the tube is $c\pi$ cm^2.
Find the value of c.



c =

(iv) 拋擲兩粒正常骰子，設取得點數總和是 $\frac{c}{6}$ 的概率為 d，求 d 的值。
Two fair dice are thrown. Let d be the probability of getting the sum of scores to be $\frac{c}{6}$.
Find the value of d.

d =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
		+	Bonus score				Time
			Total score				Min. / Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $m, n > 0$ 和 $m + n = 1$。若 $\left(1+\frac{1}{m}\right)\left(1+\frac{1}{n}\right)$ 之最小值為 a，求 a 的值。

It is given that $m, n > 0$ and $m + n = 1$.

If the minimum value of $\left(1+\frac{1}{m}\right)\left(1+\frac{1}{n}\right)$ is a, find the value of a.

$a =$

(ii) 方程 $x^2 - (10 + a)x + 25 = 0$ 的根是 $x^2 + bx = 5$ 的根的平方，求 b 的正數值。

If the roots of the equation $x^2 - (10 + a)x + 25 = 0$ are the square of the roots of the equation $x^2 + bx = 5$, find the positive value of b.

$b =$

(iii) 若 $(xy - 2)^{b-1} + (x - 2y)^{b-1} = 0$ 及 $c = x^2 + y^2 - 1$，求 c 的值。

If $(xy - 2)^{b-1} + (x - 2y)^{b-1} = 0$ and $c = x^2 + y^2 - 1$, find the value of c.

$c =$

(iv) 若 $f(x)$ 是一二次多項式，$f(f(x)) = x^4 - 2x^2$ 及 $d = f(c)$, 求 d 的值。

If $f(x)$ is a polynomial of degree two, $f(f(x)) = x^4 - 2x^2$ and $d = f(c)$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
			+ Bonus score				Time
			Total score				Min. Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 a 為實數及 $2a^3 + a^2 - 275 = 0$，求 a 的值。
If a is a real number and $2a^3 + a^2 - 275 = 0$, find the value of a.

$a =$

(ii) 若 $3^2 \cdot 3^5 \cdot 3^8 \cdots 3^{3b-1} = 27^a$，求 b 的值。
Find the value of b if $3^2 \cdot 3^5 \cdot 3^8 \cdots 3^{3b-1} = 27^a$.

$b =$

(iii) 若 $\log_b\left(b^c - 8\right) = 2 - c$，求 c 的值。
Find the value of c if $\log_b\left(b^c - 8\right) = 2 - c$.

$c =$

(iv) 若 $[(4^c)^c]^c = 2^d$，求 d 的值。
If $[(4^c)^c]^c = 2^d$, find the value of d.

$d =$

FOR OFFICIAL USE

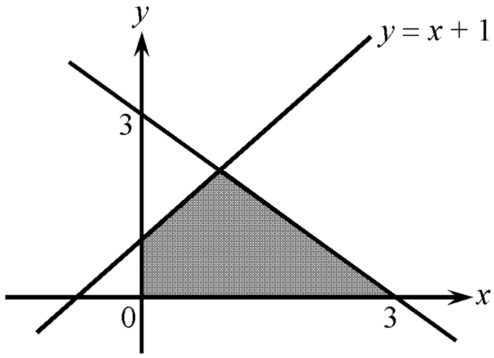
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 圖中陰影部分面積是 a，求 a 的值。
In the figure, the area of the shaded region is a. Find the value of a.

$a =$



(ii) 若 $8^b = 4^a - 4^3$，求 b 的值。
If $8^b = 4^a - 4^3$, find the value of b.

$b =$

(iii) 已知 c 是方程式 $x^2 - 100b + \frac{10000}{x^2} = 0$ 之正根，求 c 的值。

Given that c is the positive root of the equation $x^2 - 100b + \frac{10000}{x^2} = 0$,

find the value of c.

$c =$

(iv) 若 $d = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \cdots + \frac{1}{(c-1)\times c}$，求 d 的值。

If $d = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \cdots + \frac{1}{(c-1)\times c}$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 同時投擲四顆骰子。設取得最小一半骰子的結果為偶數的概率為 a，求 a 的值。
Four fair dice are thrown. Let a be the probability of getting at least half of the outcome of the dice to be even. Find the value of a.

$a =$

(ii) 已知 $\mathrm{f}(x)=\frac{3}{8}x^2(81)^{-\frac{1}{x}}$ 和 $g(x) = 4\log_{10}(14x) - 2\log_{10}49$。
求 $b = \mathrm{f}\{g[16(1-a)]\}$ 的值。
It is given that $\mathrm{f}(x)=\frac{3}{8}x^2(81)^{-\frac{1}{x}}$ and $g(x) = 4\log_{10}(14x) - 2\log_{10}49$.
Find the value of $b = \mathrm{f}\{g[16(1-a)]\}$.

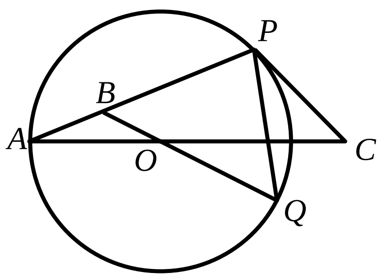
$b =$

(iii) 設 $c=\frac{1}{b^2-1}+\frac{1}{(2b)^2-1}+\frac{1}{(3b)^2-1}+\cdots+\frac{1}{(10b)^2-1}$，求 c 的值。
提示：$\frac{1}{x^2-1}=\frac{1}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)$
Let $c=\frac{1}{b^2-1}+\frac{1}{(2b)^2-1}+\frac{1}{(3b)^2-1}+\cdots+\frac{1}{(10b)^2-1}$, find the value of c.
Hint: $\frac{1}{x^2-1}=\frac{1}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)$

$c =$

(iv) 在下圖中，PC 是圓(圓心為 O)的切綫，切點在 P。ΔABO 是等腰三角形，$AB = OB$，$\angle PCO = c$ 及 $d = \angle QPC$，其中 c、d 為弧度。求 d 的值。(取$\pi=\frac{22}{7}$)
In the following diagram, PC is a tangent to the circle (centre O) at the point P, and ΔABO is an isosceles triangle, $AB = OB$, $\angle PCO = c$ and $d = \angle QPC$, where c and d are radian measures. Find the value of d. (Take $\pi=\frac{22}{7}$)

$d =$



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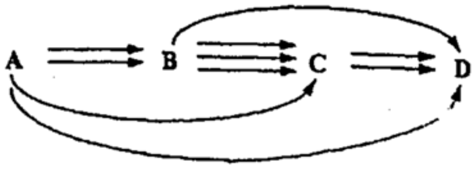
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1995-96)
Spare Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 下圖中，由 A 到 D 共有 a 條路徑，求 a 的值。
From the following figure, determine the number of routes a from A to D.



$a =$

(ii) 若 $\sin(2b° + 2a°) = \cos(6b° - 16°)$，其中 $0 < b < 90$，求 b 的值。
If $\sin(2b° + 2a°) = \cos(6b° - 16°)$, where $0 < b < 90$, find the value of b.

$b =$

(iii) 直綫 $(bx - 6y + 3) + k(x - y + 1) = 0$ 經過 $P(c, m)$，其中 k 是任何實數，求 c 的值。
The lines $(bx - 6y + 3) + k(x - y + 1) = 0$, where k is any real constant, pass through a fixed point $P(c, m)$, find the value of c.

$c =$

(iv) 已知 $d^2 - c = 257 \times 259$。求 d 的正值。
It is known that $d^2 - c = 257 \times 259$. Find the positive value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 一籃子雞蛋的數目為 a，分三輪派發。第一輪派出一半另半枚，第二輪派出剩下的一半另半枚，第三輪又派出剩下的一半另半枚。籃子中的雞蛋便全部派光，求 a 的值。
The number of eggs in a basket was a. Eggs were given out in three rounds. In the first round half of egg plus half an egg were given out. In the second round, half of the remaining eggs plus half an egg were given out. In the third round, again, half of the remaining eggs plus half an egg were given out. The basket then became empty.
Find the value of a.

$a =$

(ii) 若 $p - q = 2; p - r = 1$ 及 $b = (r - q)[(p - q)^2 + (p - q)(p - r) + (p - r)^2]$，求 b 的值。
If $p - q = 2; p - r = 1$ and $b = (r - q)[(p - q)^2 + (p - q)(p - r) + (p - r)^2]$.
Find the value of b.

$b =$

(iii) 若 n 是一正整數，$m^{2n} = 2$ 及 $c = 2m^{6n} - 4$，求 c 的值。
If n is a positive integer, $m^{2n} = 2$ and $c = 2m^{6n} - 4$, find the value of c.

$c =$

(iv) 若 r, s, t, u 是正整數及 $r^5 = s^4, t^3 = u^2, t - r = 19$ 及 $d = u - s$，求 d 的值。
If r, s, t, u are positive integers and $r^5 = s^4, t^3 = u^2, t - r = 19$ and $d = u - s$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1995-96)

Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若方程 $ax^2 - mx + 1996 = 0$ 的兩個不等根是質數，求 a 的值。
If the two distinct roots of the equation $ax^2 - mx + 1996 = 0$ are primes, find the value of a.

$a =$

(ii) 六位數 $111aaa$ 是兩個連續正整數 b 和 $b+1$ 之積，求 b 的值。
A six-digit figure $111aaa$ is the product of two consecutive positive integers b and $b+1$, find the value of b.

$b =$

(iii) 若 p, q, r 是非零實數，$p^2 + q^2 + r^2 = 1$，
$p\left(\frac{1}{q}+\frac{1}{r}\right)+q\left(\frac{1}{r}+\frac{1}{p}\right)+r\left(\frac{1}{p}+\frac{1}{q}\right)+3=0$，及 $c = p + q + r$，求 c 的最大值。
If p, q, r are non-zero real numbers;
$p^2 + q^2 + r^2 = 1$, $p\left(\frac{1}{q}+\frac{1}{r}\right)+q\left(\frac{1}{r}+\frac{1}{p}\right)+r\left(\frac{1}{p}+\frac{1}{q}\right)+3=0$ and $c = p + q + r$,
find the largest value of c.

$c =$

(iv) 若 7^{14} 之個位是 d，求 d 的值。
If the units digit of 7^{14} is d, find the value of d.

$d =$

FOR OFFICIAL USE

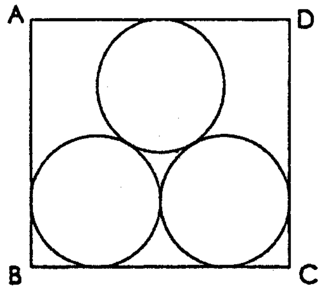
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

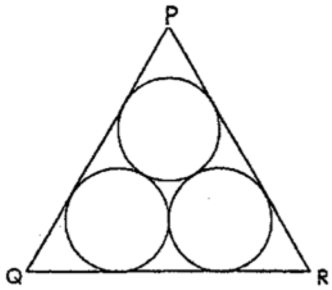
In this question, all unnamed circles are unit circles.
在本題內，所有不命名的圓皆是單位圓。

(i) 若矩形 $ABCD$ 的面積是 $a+4\sqrt{3}$，求 a 的值。
If the area of the rectangle $ABCD$ is $a+4\sqrt{3}$, find the value of a.



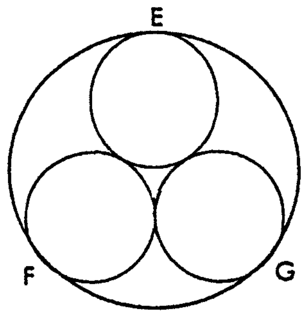
$a =$

(ii) 若等邊三角形 PQR 的面積是 $6+b\sqrt{3}$，求 b 的值。
If the area of the equilateral triangle PQR is $6+b\sqrt{3}$, find the value of b.



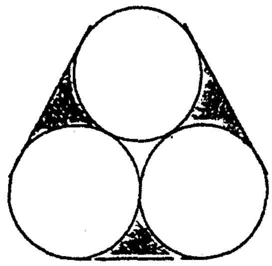
$b =$

(iii) 若圓 EFG 的面積是 $\frac{(c+4\sqrt{3})\pi}{3}$，求 c 的值。
If the area of the circle EFG is $\frac{(c+4\sqrt{3})\pi}{3}$,
find the value of c.



$c =$

(iv) 若下圖所有直綫皆是兩個圓的公切綫，且陰影部份的面積是 $6+d\pi$，求 d 的值。
If all the straight lines in the diagram below are common tangents to the two circles, and the area of the shaded part is $6+d\pi$, find the value of d.



$d =$

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Score for accuracy [ ] × Mult. factor for speed [ ] = [ ] Team No. [ ]

\+ Bonus score [ ] Time [ ] [ ]

Total score [ ] Min. Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $(1995)^a + (1996)^a + (1997)^a$ 能被 10 整除，求 a 的最小可能整數值。
If $(1995)^a + (1996)^a + (1997)^a$ is divisible by 10,
find the least possible integral value of a.

$a =$

(ii) 若 $(x^2 + y^2)^2 \le b(x^4 + y^4)$ 對任意實數 x 和 y 都成立，求 b 的最小可能整數值。
If the expression $(x^2 + y^2)^2 \le b(x^4 + y^4)$ holds for all real values of x and y,
find the least possible integral value of b.

$b =$

(iii) 若 $c = 1996\times19971997 - 1995\times19961996$，求 c 的值。
If $c = 1996\times19971997 - 1995\times19961996$, find the value of c.

$c =$

(iv) 若

$$d=\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{60}\right)+\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\cdots+\frac{2}{60}\right)+\left(\frac{3}{4}+\frac{3}{5}+\cdots+\frac{3}{60}\right)+\cdots+\left(\frac{58}{59}+\frac{58}{60}\right)+\frac{59}{60}，$$

求 d 的值。
Find the sum d where

$$d=\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{60}\right)+\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\cdots+\frac{2}{60}\right)+\left(\frac{3}{4}+\frac{3}{5}+\cdots+\frac{3}{60}\right)+\cdots+\left(\frac{58}{59}+\frac{58}{60}\right)+\frac{59}{60}.$$

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]   Team No. [ ]

+ Bonus score [ ]   Time [ ] [ ]

Total score [ ]   Min.   Sec.

Hong Kong Mathematics Olympiad (1995-96)
Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $3\times4\times5\times6 = 19^2 - 1$
$4\times5\times6\times7 = 29^2 - 1$
$5\times6\times7\times8 = 41^2 - 1$
$6\times7\times8\times9 = 55^2 - 1$
若 $a^2 = 1000\times1001\times1002\times1003 + 1$，求 a 的值。

It is given that $3\times4\times5\times6 = 19^2 - 1$
$4\times5\times6\times7 = 29^2 - 1$
$5\times6\times7\times8 = 41^2 - 1$
$6\times7\times8\times9 = 55^2 - 1$
If $a^2 = 1000\times1001\times1002\times1003 + 1$, find the value of a.

$a =$

(ii) 設 $f(x) = x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$。
當 $f(x^{10})$ 除以 $f(x)$，餘數是 b。求 b 的值。
Let $f(x) = x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$.
When $f(x^{10})$ is divided by $f(x)$, the remainder is b. Find the value of b.

$b =$

(iii) 分數 $\frac{p}{q}$ 已化成最簡形式。若 $\frac{7}{10} < \frac{p}{q} < \frac{11}{15}$，當中 q 是最小可能正整數，且 $c = pq$，求 c 的值。
The fraction $\frac{p}{q}$ is in its simplest form. If $\frac{7}{10} < \frac{p}{q} < \frac{11}{15}$ where q is the smallest possible positive integer and $c = pq$. Find the value of c.

$c =$

(iv) 若正整數 d 除以 7，餘數是 1；除以 5 餘數是 2；除以 3 餘數是 2。求 d 的最小可能值。
A positive integer d when divided by 7 will have 1 as its remainder; when divided by 5 will have 2 as its remainder and when divided by 3 will have 2 as its remainder.
Find the least possible value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]　　Team No. [ ]

\+ Bonus score [ ]　　Time [ ] [ ]

Total score [ ]　　Min.　Sec.