Hong Kong Mathematics Olympiad (1995– 96)
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。　　　　　　　時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 已知 $4^{x-3} = 8^{x-2}$，求 x 的值。

   Find the value of x if $4^{x-3} = 8^{x-2}$.

2. 已知 $f\left(\dfrac{1+x}{x}\right) = \dfrac{x^2+1}{x^2} + \dfrac{1}{x}$，求 $f(x^3)$ 的值。

   If $f\left(\dfrac{1+x}{x}\right) = \dfrac{x^2+1}{x^2} + \dfrac{1}{x}$, find $f(x^3)$.

3. 考慮 $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$，求 $100!$ 的末尾 0 的數目。

   By considering $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$, find the number of trailing zeros of $100!$.

4. 能滿足不等式 $n^{200} < 5^{300}$ 的最大整數 n 是多少？

   What is the largest integral value n that satisfies the inequality $n^{200} < 5^{300}$?

5. 三個面值 $0.1、$3 和 $5 和郵票共有 110 個，這組郵票共值 $100。求 $3 郵票的數目。

   A set of 110 stamps of the denominations of $0.1, $3, $5 worth $100 in total.

   Find the number of $3 stamps in the set of stamps.

6. 對於任何數值 m，直綫 $y = mx + 2m + 2$ 必經一定點 P。求 P 之座標。

   For any value of m, a straight line $y = mx + 2m + 2$ passes through a fixed point P. Find the coordinates of P.

7. 若各數字可重複選用，從數字 $4, 5, 6, 7, 8, 9$ 中，可組成多少個 3 位數？

   How many 3-digit numbers can be made from the figures 4, 5, 6, 7, 8, 9 when repetitions are allowed?

8. 將 $2.3\dot{1}$ 以分數表示。

   Express $2.3\dot{1}$ as a fraction.

9. 若 x 及 y 為正整數及 $x - y = 5$，求 $x^2 - y^2 + 5$ 之最小值。

   If x and y are positive integers and $x - y = 5$, find the least value of $x^2 - y^2 + 5$.

10. 因式分解 $x^2(y-z) + y^2(z-x) + z^2(x-y)$。

    Factorize $x^2(y-z) + y^2(z-x) + z^2(x-y)$.

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1.　圖中二次函數曲綫 $y=f(x)$ 切 x-軸於點 $(1, 0)$ 和 $(5, 0)$，及 y-軸於點 $(0, -10)$。求 p 的值。

In the figure, the quadratic curve $y=f(x)$ cuts the x-axis at the two points $(1, 0)$ and $(5, 0)$ and the y-axis at the point $(0, -10)$. Find the value of p.
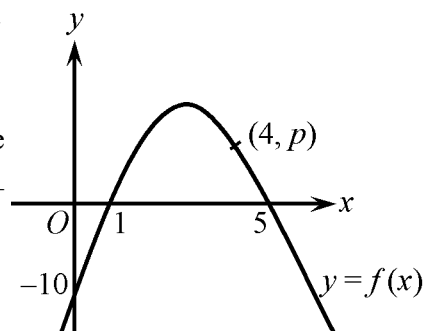
2.　在圖中 O 是圓錐體底部的圓心；A、B、C 及 O 躺於同一平面上。若螞蟻在圓錐曲面上由 A 走到 B，找出由 A 到 B 的最短路綫的長度。

In the figure, O is the centre of the base circle of a cone and the points A, B, C and O lie in the same plane. An ant walks from A to B on the surface of the cone. Find the length of the shortest path from A to B.
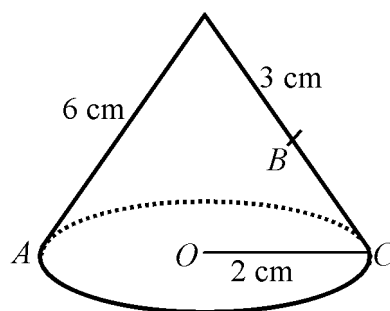
3.　當一疊面值 7020 元的十元紙幣被等分給 x 人時，餘下 650 元。把剩下的 650 元換成 5 元硬幣再等分給 x 人時，餘下 195 元。求 x 的值。

When a sum of $7020, in the form of ten-dollar notes, is divided equally among x persons, $650 remains. When this sum $650 is changed to five-dollar coins and then divided equally among the x persons, $195 remains. Find the value of x.

4.　射擊比賽中，根據統計紀錄，A 每 5 發射擊，有一發落空，B 每 4 發射擊則有一發落空，而 C 每 3 發射擊則有一發落空。

若 A、B、C 同時各發一槍，求 A、B 命中而 C 落空的概率。

In a shooting competition, according to statistics, A misses one in every 5 shoots, B misses one in every 4 shoots and C misses one in every 3 shoots. Find the probability of obtaining successful shoots by A, B but not C.

5.　Given that $\dfrac{1}{n(n+1)}=\dfrac{1}{n}-\dfrac{1}{n+1}$, find the value of $\dfrac{1}{2\times3}+\dfrac{1}{3\times4}+\cdots+\dfrac{1}{99\times100}$.

已知 $\dfrac{1}{n(n+1)}=\dfrac{1}{n}-\dfrac{1}{n+1}$，求 $\dfrac{1}{2\times3}+\dfrac{1}{3\times4}+\cdots+\dfrac{1}{99\times100}$ 之值。

6.   若三位數 A 加上 3，新數的三個數字之和為原先 A 的三位數之和的三分之一。求所有這些可能的數目 A 的總和。

If 3 is added to a 3-digit number A, the sum of the digits of the new number is $\dfrac{1}{3}$ of the value of the sum of digits of the original number A. Find the sum of all such possible numbers A.

7.   圖中每個小正方形的邊長為 1 單位。求圖中所有可能組成之長方形(包括正方形)的面積之和。

In the figure, the side of each smaller square is 1 unit long. Find the sum of the area of all possible rectangles (squares included) that can be formed in the figure.
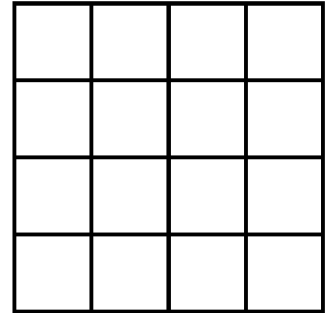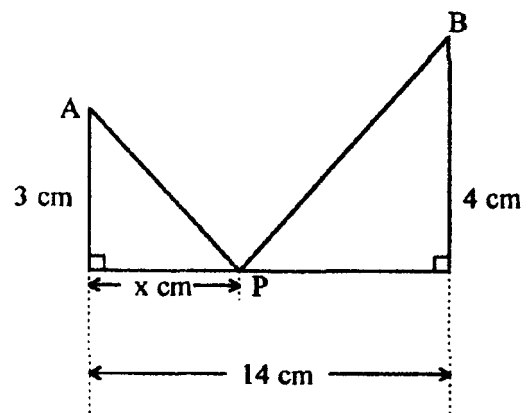
8.   若質數 a、b 為二次方程 $x^2 - 21x + t = 0$ 的根，求 $\left(\dfrac{b}{a} + \dfrac{a}{b}\right)$ 的值。

If prime numbers a, b are the roots of the quadratic equation $x^2 - 21x + t = 0$, find the value of $\left(\dfrac{b}{a} + \dfrac{a}{b}\right)$.

9.   求 x 的值使得圖中路綫 APB 的長度最小。

Find the value of x such that the length of the path APB in the figure is the smallest .

10.   求總數 $1^2 + 2^2 + 3^2 + 4^2 + \cdots + 123456789^2$ 的個位數。

Find the units digit of the sum $1^2 + 2^2 + 3^2 + 4^2 + \cdots + 123456789^2$.

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