Hong Kong Mathematics Olympiad (1996-97)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ 及 $\frac{2}{a}-\frac{3}{u}=6$ 為 a 與 u 的聯立方程。求 a 的解。

Given that $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ and $\frac{2}{a}-\frac{3}{u}=6$ are simultaneous equations in a and u. Solve for a.

$a =$

(ii) 方程 $px+qy+bz=1$ 的根分別為 $(0, 3a, 1)$、$(9a, -1, 2)$ 和 $(0, 3a, 0)$。求係數 b 的值。

Three solutions of the equation $px+qy+bz=1$ are $(0, 3a, 1)$, $(9a, -1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b.

$b =$

(iii) 若 $y=mx+c$ 的圖像經過 $(b+4, 5)$ 及 $(-2, 2)$ 兩點。求 c 的值。

Find the value of c so that the graph of $y=mx+c$ passes through the two points $(b+4, 5)$ and $(-2, 2)$.

$c =$

(iv) 不等式 $x^2+5x-2c\le 0$ 的解為 $d\le x\le 1$。求 d 的值。

The solution of the inequality $x^2+5x-2c\le 0$ is $d\le x\le 1$. Find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

+ Bonus score

Total score

Team No.

Time Min. Sec.

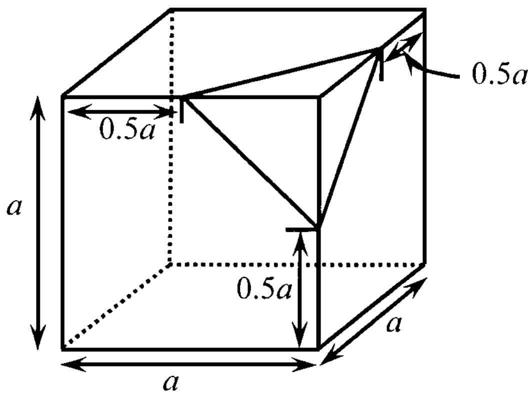
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 考慮：$\frac{1^2}{1}=1$，$\frac{1^2+2^2}{1+2}=\frac{5}{3}$，$\frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}$，$\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=3$，
求 a 的值使得 $\frac{1^2+2^2+\cdots+a^2}{1+2+\cdots+a}=\frac{25}{3}$。
By considering: $\frac{1^2}{1}=1, \frac{1^2+2^2}{1+2}=\frac{5}{3}, \frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}, \frac{1^2+2^2+3^2+4^2}{1+2+3+4}=3$,
find the value of a such that $\frac{1^2+2^2+\cdots+a^2}{1+2+\cdots+a}=\frac{25}{3}$.

$a =$

(ii) 如圖所示，從邊長為 a cm 的正立方體的一角割出一個三角錐體。
若三角錐體的體積為 b cm^3，求 b 的值。
A triangular pyramid is cut from a corner of a cube with side length a cm as the figure shown. If the volume of the pyramid is b cm^3, find the value of b.

$b =$



(iii) 若對於所有實數 x，x^2+cx+b 不小於 0，求 c 的最大值。
If the value of x^2+cx+b is not less than 0 for all real number x, find the maximum value of c.

$c =$

(iv) 若 1997^{1997} 的個位數為 $c-d$，求 d 的值。
If the units digit of 1997^{1997} is $c-d$, find d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) a、b、c 和 d 的平均值為 8。若 a、b、c、d 和 P 的平均值為 P，求 P 的值。
The average of a, b c and d is 8. If the average of a, b, c, d and P is P, find the value of P.

P =

(ii) 若直綫 $2x + 3y + 2 = 0$ 和 $Px + Qy + 3 = 0$ 互相平行，求 Q 的值。
If the lines $2x + 3y + 2 = 0$ and $Px + Qy + 3 = 0$ are parallel, find the value of Q.

Q =

(iii) 若等邊三角形的周界和面積分別為 Q cm 和 $\sqrt{3}R$ cm^2。求 R 的值。
The perimeter and the area of an equilateral triangle are Q cm and $\sqrt{3}R$ cm^2 respectively. Find the value of R.

R =

(iv) 若 $(1 + 2 + \ldots + R)^2 = 1^2 + 2^2 + \ldots + R^2 + S$，求 S 的值。
If $(1 + 2 + \ldots + R)^2 = 1^2 + 2^2 + \ldots + R^2 + S$, find the value of S.

S =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若正 n 邊形的內角為 140°，求 n 的值。
If each interior angle of a n-sided regular polygon is 140°, find the value of n.

$n =$

(ii) 若不等式 $2x^2 - nx + 9 < 0$ 的解為 $k < x < b$，求 b 的值。
If the solution of the inequality $2x^2 - nx + 9 < 0$ is $k < x < b$, find the value of b.

$b =$

(iii) 若 $cx^3 - bx + x - 1$ 除以 $x + 1$，餘數為 -7，求 c 的值。
If $cx^3 - bx + x - 1$ is divided by $x + 1$, the remainder is -7, find the value of c.

$c =$

(iv) 若 $x + \frac{1}{x} = c$ 和 $x^2 + \frac{1}{x^2} = d$，求 d 的值。
If $x + \frac{1}{x} = c$ and $x^2 + \frac{1}{x^2} = d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 一直徑為 a 的半球體的體積為 18π cm^3，求 a 的值。
The volume of a hemisphere with diameter a cm is 18π cm^3, find the value of a.

$a =$

(ii) 若 $\sin 10a° = \cos(360° - b°)$ 和 $0 < b < 90$，求 b 的值。
If $\sin 10a° = \cos(360° - b°)$ and $0 < b < 90$, find the value of b.

$b =$

(iii) 一三角形是由 x-軸、y-軸和直綫 $bx + 2by = 120$ 所組成。
若所包圍之三角形的面積為 c，求 c 的值。
The triangle is formed by the x-axis and y-axis and the line $bx + 2by = 120$.
If the bounded area of the triangle is c, find the value of c.

$c =$

(iv) 若方程式 $x^2 - (c + 2)x + (c + 1) = 0$ 兩根之差為 d，求 d 的值。
If the difference of the two roots of the equation $x^2 - (c + 2)x + (c + 1) = 0$ is d, find the value of d.

$d =$

FOR OFFICIAL USE

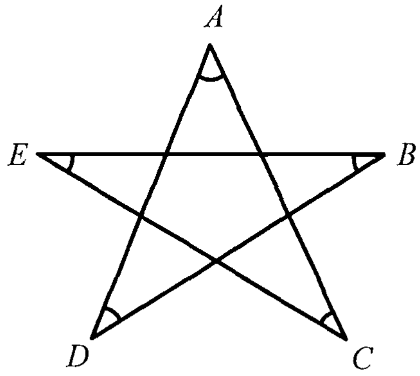
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 圖中，$\angle A + \angle B + \angle C + \angle D + \angle E = a°$。求 a 的值。
In the diagram, $\angle A + \angle B + \angle C + \angle D + \angle E = a°$. Find the value of a.

$a =$



(ii) 代數式 $x^6 + x^6 + x^6 + \ldots + x^6$ 有 x 項及其總和為 x^b。求 b 的值。
There are x terms in the algebraic expression $x^6 + x^6 + x^6 + \ldots + x^6$ and its sum is x^b. Find the value of b.

$b =$

(iii) 若 $1 + 3 + 3^2 + 3^3 + \ldots + 3^8 = \frac{3^c - 1}{2}$，求 c 的值。
If $1 + 3 + 3^2 + 3^3 + \ldots + 3^8 = \frac{3^c - 1}{2}$, find the value of c.

$c =$

(iv) 從 16 張寫上 1 至 16 的咭紙中隨意抽出一張，若果抽出的號碼是一個完全平方數的概率為 $\frac{1}{d}$，求 d 之值。
16 cards are marked from 1 to 16 and one is drawn at random.
If the chance of it being a perfect square number is $\frac{1}{d}$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若數列 1、$6+2a$、$10+5a$、$\cdots$ 是一算術級數，求 a 的值。

If the sequence $1, 6+2a, 10+5a, \cdots$ forms an A.P., find the value of a.

$a =$

(ii) 若 $(0.0025\times 40)^b = \frac{1}{100}$，求 b 的值。

If $(0.0025\times 40)^b = \frac{1}{100}$, find the vale of b.

$b =$

(iii) 若 c 為正整數及 $c^3+3c+\frac{3}{c}+\frac{1}{c^3}=8$，求 c 的值。

If c is an integer and $c^3+3c+\frac{3}{c}+\frac{1}{c^3}=8$, find the value of c.

$c =$

(iv) 若將 5 個女孩排成一列，共有 d 個不同方法。求 d 的值。
There are d different ways for arranging 5 girls in a row. Find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 設 m 為滿足不等式 $14x - 7(3x - 8) < 4(25 + x)$的整數。求 m 的最小值。
Let m be an integer satisfying the inequality: $14x - 7(3x - 8) < 4(25 + x)$.
Find the least value of m.

$m =$

(ii) 已知 $f(x)=\frac{1}{3}x^3-2x^2+\frac{2}{3}x^3+3x^2+5x+7-4x$。若 $f(-2) = b$，求 b 的值。
It is given that $f(x)=\frac{1}{3}x^3-2x^2+\frac{2}{3}x^3+3x^2+5x+7-4x$.
If $f(-2) = b$, find the value of b.

$b =$

(iii) 已知 $\log\frac{x}{2}=0.5$ 及 $\log\frac{y}{5}=0.1$。若 $\log xy = c$，求 c 的值。
It is given that $\log\frac{x}{2}=0.5$ and $\log\frac{y}{5}=0.1$. If $\log xy = c$, find the value of c.

$c =$

(iv) d、e 及 f 為三個小於 10 之質數且滿足兩個條件 $d + e = f$ 及 $d < e$。求 d 的值。
Three prime numbers d, e and f which are all less than 10, satisfy the two conditions $d + e = f$ and $d < e$. Find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
			+ Bonus score				Time
			Total score				Min. / Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $a = 103\times97\times10009$，求 a 的值。
It is given that $a = 103\times97\times10009$, find the value of a.

$a =$

(ii) 已知 $1 + x + x^2 + x^3 + x^4 = 0$。若 $b = 2 + x + x^2 + x^3 + x^4 + \ldots + x^{1989}$，求 b 的值。
It is given that $1 + x + x^2 + x^3 + x^4 = 0$.
If $b = 2 + x + x^2 + x^3 + x^4 + \ldots + x^{1989}$, find the value of b.

$b =$

(iii) 已知 m 及 n 為兩個不大於 10 的自然數。
若 c 為 m 及 n 滿足方程 $mx = n$ 之組數，其中 $\frac{1}{4} < x < \frac{1}{3}$。求 c 的值。
It is given that m and n are two natural numbers and both are not greater than 10. If c is the number of pairs of m and n satisfying the equation $mx = n$, where $\frac{1}{4} < x < \frac{1}{3}$, find the value of c.

$c =$

(iv) 設 x 及 y 為實數且定義運算*為 $x*y = px^y + q + 1$。已知 $1*2 = 869$ 及 $2*3 = 883$。若 $2*9 = d$，求 d 的值。
Let x and y be real numbers and define the operation * as $x*y = px^y + q + 1$.
It is given that $1*2 = 869$ and $2*3 = 883$. If $2*9 = d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+ Bonus score				Time		
			Total score					Min.	Sec.

Hong Kong Mathematics Olympiad (1996-97)
Final Event 5 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 a 是 5 的正倍數，且被 3 除時餘 1，求 a 之最小可能數值。
If a is a positive multiple of 5, which gives remainder 1 when divided by 3, find the smallest possible value of a.

$a =$

(ii) 若 $x^3 + 6x^2 + 12x + 17 \equiv (x + 2)^3 + b$，求 b 的值。
If $x^3 + 6x^2 + 12x + 17 \equiv (x + 2)^3 + b$, find the value of b.

$b =$

(iii) 若 c 是一兩位正整數，其兩位之和是 10 而兩位之積是 25。求 c 的值。
If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digit is 25, find the value of c.

$c =$

(iv) 設 S_1、S_2、⋯、S_{10} 是一個由正整數組成的 A.P.之首 10 項。
若 $S_1 + S_2 + \cdots + S_{10} = 55$ 及 $(S_{10} - S_8) + (S_9 - S_7) + \cdots + (S_3 - S_1) = d$。求 d 的值。
Let $S_1, S_2, \cdots, S_{10}$ be the first ten terms of an A.P., which consists of positive integers.
If $S_1 + S_2 + \cdots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \cdots + (S_3 - S_1) = d$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

Time [ ] Min. [ ] Sec.

Total score [ ]

Hong Kong Mathematics Olympiad (1996-97)
Final Event (Spare Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) E 是平行四邊形 $ABCD$ 其中一條邊 CD 的中點。若三角形 ADE 與平行四邊形 $ABCD$ 面積的比等於 $1 : a$，求 a 的值。
$ABCD$ is a parallelogram and E is the midpoint of CD. If the ratio of the area of the triangle ADE to the area of the parallelogram $ABCD$ is $1 : a$, find the value of a.

$a =$

(ii) E 是平行四邊形 $ABCD$ 其中一條邊 CD 的中點，且 AE 和 BD 相交於 M；若 $DM : MB = 1 : k$，求 k 的值。
$ABCD$ is a parallelogram and E is the midpoint of CD. AE and BD meet at M. If $DM : MB = 1 : k$, find the value of k.

$k =$

(iii) 若 5 的平方根是 2.236，以同一準確度，80 的平方根是 d。求 d 的值。
If the square root of 5 is approximately 2.236, the square root of 80 with the same precision is d. Find the value of d

$d =$

(iv) 將一個正方形的長增加 20%，同時又將它的闊減少 20%，則我們可得一個長方形。若長方形與正方形面積的比為 $1 : r$，求 r 的值。
A square is changed into a rectangle by increasing its length by 20% and decreasing its width by 20%. If the ratio of the area of the rectangle to the area of the square is $1 : r$, find the value of r.

$r =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]   Team No. [ ]

+ Bonus score [ ]   Time [ ] Min. [ ] Sec.

Total score [ ]