Hong Kong Mathematics Olympiad (1997-98)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ 及 $\frac{2}{a}-\frac{3}{u}=6$ 為 a 與 u 的聯立方程。求 a 的解。

Given that $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ and $\frac{2}{a}-\frac{3}{u}=6$ are simultaneous equations in a and u.
Solve for a.

$a =$

(ii) 方程 $px + qy + bz = 1$ 的根分別為$(0, 3a, 1)$、$(9a, –1, 2)$和$(0, 3a, 0)$。
求係數 b 的值。

Three solutions of the equation $px + qy + bz = 1$ are $(0, 3a, 1)$, $(9a, –1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b.

$b =$

(iii) 若 $y = mx + c$ 的圖像經過 $(b + 4, 5)$ 及 $(–2, 2)$ 兩點。求 c 的值。

Find the value of c so that the graph of $y = mx + c$ passes through the two points $(b + 4, 5)$ and $(–2, 2)$.

$c =$

(iv) 不等式 $x^2 + 5x – 2c \le 0$ 的解為 $d \le x \le 1$。求 d 的值。

The solution of the inequality $x^2 + 5x – 2c \le 0$ is $d \le x \le 1$. Find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy	×	Mult. factor for speed	=		Team No.	
			+	Bonus score	Time	
				Total score	Min.	Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 a 是 $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$ 的最大值，求 a 的值。

If a is the maximum value of $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$, find the value of a.

$a =$

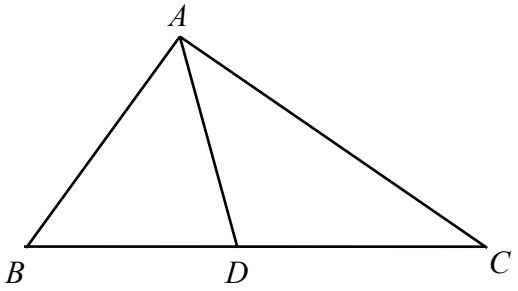
(ii) 若 $\begin{cases} x+y=2 \\ xy-z^2=a \\ b=x+y+z \end{cases}$，求 b 的值。

If $\begin{cases} x+y=2 \\ xy-z^2=a \\ b=x+y+z \end{cases}$, find the value of b.

$b =$

(iii) 在圖中，$BD = b$ cm，$DC = c$ cm，且 ΔABD 的面積 $= \frac{1}{3} \times \Delta ABC$ 的面積，求 c 的值。

In the figure, $BD = b$ cm, $DC = c$ cm and area of $\Delta ABD = \frac{1}{3} \times$ area of ΔABC, find the value of c.



$c =$

(iv) 設 d 為 $500 + c$ 的正因數的數目，求 d 的值。

Suppose d is the number of positive factors of $500 + c$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 2 (Individual)

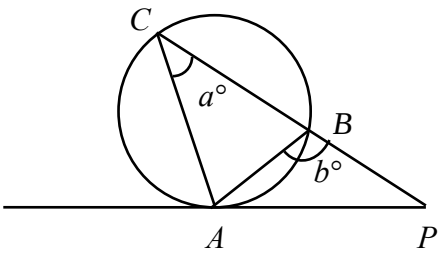
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $A(1, 3)$、$B(5, 8)$及 $C(29, a)$共綫，求 a 的值。
If $A(1, 3)$, $B(5, 8)$ and $C(29, a)$ are collinear, find the value of a.

$a =$

(ii) 在圖中，PA 切圓 ABC 於 A。PBC 為一直綫、$AB = BP$、$\angle ACB = a°$。
若 $\angle ABP = b°$，求 b 的值。
In the figure, PA touches the circle ABC at A, PBC is a straight line, $AB = PB$, $\angle ACB = a°$. If $\angle ABP = b°$, find the value of b .

$b =$



(iii) 若 c 為二次函數 $y = x^2 + 4x + b$ 之最小值，求 c 的值。
If c is the minimum value of the quadratic function $y = x^2 + 4x + b$, find the value of c.

$c =$

(iv) 若 $d = 1 - 2 + 3 - 4 + \ldots - c$，求 d 的值。
If $d = 1 - 2 + 3 - 4 + \ldots - c$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\{p, q\} = q \times a + p$ 且 $\{2, 5\} = 52$，求 a 的值。
If $\{p, q\} = q \times a + p$ and $\{2, 5\} = 52$, find the value of a.

$a =$

(ii) 若數列 a，$\frac{37}{2}$，b 為一等差數列，求 b 的值。
If $a, \frac{37}{2}, b$ is an arithmetic progression, find the value of b.

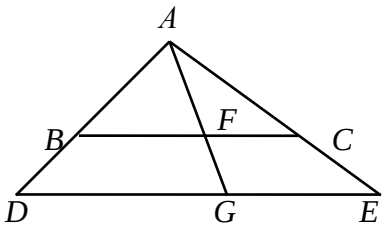
$b =$

(iii) 若 $b^2 - c^2 = 200$ 及 $c > 0$，求 c 的值。
If $b^2 - c^2 = 200$ and $c > 0$, find the value of c.

$c =$

(iv) 在圖中，已知 $BC // DE$、$BC : DE = 10 : c$ 及 $AF : FG = 20 : d$，求 d 的值。
Given that in the figure, $BC // DE$, $BC : DE = 10 : c$ and $AF : FG = 20 : d$, find the value of d.

$d =$



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Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $\frac{10x-3y}{x+2y}=2$ 且 $p=\frac{y+x}{y-x}$，求 p 的值。
Given that $\frac{10x-3y}{x+2y}=2$ and $p=\frac{y+x}{y-x}$, find the value of p.

$p =$

(ii) 已知 $a \neq b$ 且 $ax = bx$。若 $p + q = 19(a - b)^x$，求 q 的值。
Given that $a \neq b$ and $ax = bx$. If $p + q = 19(a - b)^x$, find the value of q.

$q =$

(iii) 已知 q 個連續數之和為 222，其中最大的是 r，求 r 的數值。
Given that the sum of q consecutive numbers is 222, and the largest of these consecutive numbers is r, find the value of r.

$r =$

(iv) 若 $\tan^2(r + s)° = 3$ 且 $0 \leq r + s \leq 90$，求 s 的值。
If $\tan^2(r + s)° = 3$ and $0 \leq r + s \leq 90$, find the value of s.

$s =$

FOR OFFICIAL USE

Score for accuracy	×	Mult. factor for speed	=		Team No.	
			+	Bonus score	Time	
				Total score	Min.	Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若方程 $5x^2 + ax - 2 = 0$ 的根的和為它的根的積的兩倍，求 a 的值。
If the sum of roots of $5x^2 + ax - 2 = 0$ is twice the product of its roots, find the value of a.

$a =$

(ii) 已知 $y = ax^2 - bx - 13$ 穿過(3, 8)，求 b 的值。
Given that $y = ax^2 - bx - 13$ passes through (3, 8), find the value of b.

$b =$

(iii) 若有 c 種排法把 b 位女孩排成一圓，求 c 的值。
If there are c ways of arranging b girls in a circle, find the value of c.

$c =$

(iv) 若 $\frac{c}{4}$ 條直綫和 3 個圓畫於一白紙上，且它們的最多交點數量為 d，求 d 的值。
If $\frac{c}{4}$ straight lines and 3 circles are drawn on a paper, and d is the largest numbers of points of intersection, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
				+ Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1997-98)
Sample Event (Group)

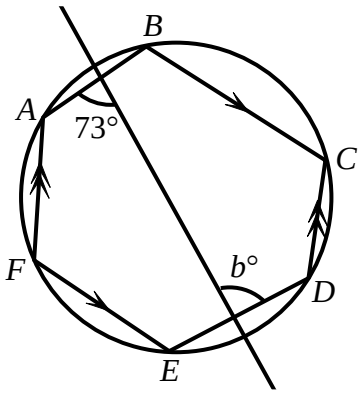
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 a 是最小的正整數被 3 除時餘 1 而能被 5 整除，求 a 的值。
If a is the smallest positive integer which gives remainder 1 when divided by 3 and is a multiple of 5, find the value of a.

$a =$

(ii) 下圖中，$FA//DC$ 及 $FE//BC$。求 b 的值。
In the following diagram, $FA//DC$ and $FE//BC$. Find the value of b.

$b =$



(iii) 若 c 是一兩位正整數，其兩位之和是 10 而兩位之積是 25，求 c 的值。
If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digit is 25, find the value of c.

$c =$

(iv) 若 $S_1, S_2, \ldots, S_{10}$ 是一由正整數組成的 A.P.的頭十項使得
$S_1 + S_2 + \ldots + S_{10} = 55$ 及 $(S_{10} - S_8) + (S_9 - S_7) + \ldots + (S_3 - S_1) = d$。求 d 的值。
If $S_1, S_2, \ldots, S_{10}$ are the first ten terms of an A.P. consisting of positive integers such that $S_1 + S_2 + \ldots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \ldots + (S_3 - S_1) = d$,
find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
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					Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

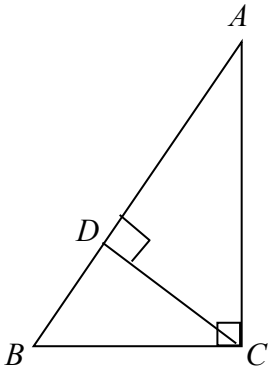
(i) 若扇形面積 $s = 4$ cm^2、扇形半徑 $r = 2$ cm 及扇形的弧長 $A = p$ cm，求 p 的值。
If the area of a given sector $s = 4$ cm^2, the radius of this sector $r = 2$ cm and the arc length of this sector $A = p$ cm, find the value of p.

$p =$

(ii) 已知 $\dfrac{a}{2b+c} = \dfrac{b}{2c+a} = \dfrac{c}{2a+b}$ 且 $a + b + c \neq 0$。若 $q = \dfrac{2b+c}{a}$，求 q 的值。
Given that $\dfrac{a}{2b+c} = \dfrac{b}{2c+a} = \dfrac{c}{2a+b}$ and $a + b + c \neq 0$.
If $q = \dfrac{2b+c}{a}$, find the value of q.

$q =$

(iii) 設直角三角形 ABC 中，CD 是斜邊 AB 上的高，$AC = 3, DB = \dfrac{5}{2}, AD = r$，求 r 的值。
Let ABC be a right-angled triangle, CD is the altitude on AB, $AC = 3$, $DB = \dfrac{5}{2}$, $AD = r$, find the value of r.



$r =$

(iv) 若 $x^3 + px^2 + qx + 17 \equiv (x + 2)^3 + a$，求 a 的值。
If $x^3 + px^2 + qx + 17 \equiv (x + 2)^3 + a$, find the value of a.

$a =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\frac{137}{a}=0.1\dot{2}3\dot{4}$，求 a 的值。
If $\frac{137}{a}=0.1\dot{2}3\dot{4}$, find the value of a.

$a =$

(ii) 若 $b = 1999\times19981998 - 1998\times19991999 + 1$，求 b 的數值。
If $b = 1999\times19981998 - 1998\times19991999 + 1$, find the value of b.

$b =$

(iii) 若參數方程 $\begin{cases}x=\sqrt{3-t^2}\\ y=t-3\end{cases}$ 可轉換為 $x^2 + y^2 + cx + dy + 6 = 0$，求 c 及 d 的值。
If the parametric equation $\begin{cases}x=\sqrt{3-t^2}\\ y=t-3\end{cases}$ can be transformed into $x^2 + y^2 + cx + dy + 6 = 0$, find the values of c and d.

$c =$

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在ΔABC 中，$\angle ABC = 2\angle ACB$，$BC = 2AB$。若$\angle BAC = a^o$，求 a 的值。
In ΔABC, $\angle ABC = 2\angle ACB$, $BC = 2AB$. If $\angle BAC = a^o$, find the value of a.

$a =$

(ii) 已知 $x + \frac{1}{x} = \sqrt{2}$，$\frac{x^2}{x^4 + x^2 + 1} = b$，求 b 的值。
Given that $x + \frac{1}{x} = \sqrt{2}$, $\frac{x^2}{x^4 + x^2 + 1} = b$, find the value of b.

$b =$

(iii) 若方程 $x + y + 2xy = 141$ 有 c 個正整數解，求 c 的值。
If the number of positive integral root(s) of the equation $x + y + 2xy = 141$ is c, find the value of c.

$c =$

(iv) 已知 $x + y + z = 0$、$x^2 + y^2 + z^2 = 1$ 及 $d = 2(x^4 + y^4 + z^4)$，求 d 的值。
Given that $x + y + z = 0$, $x^2 + y^2 + z^2 = 1$ and $d = 2(x^4 + y^4 + z^4)$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] Min. [ ] Sec.

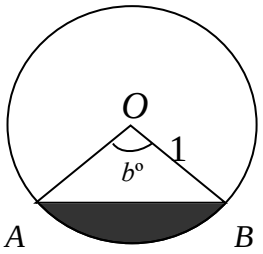
Hong Kong Mathematics Olympiad (1997-98)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $0.\dot{1}+0.0\dot{2}+0.00\dot{3}+\ldots+0.00000000\dot{9}=a$，求 a 的值。(答案以小數表示。)
If $0.\dot{1}+0.0\dot{2}+0.00\dot{3}+\ldots+0.00000000\dot{9}=a$, find the value of a.
(Give your answer in decimal)

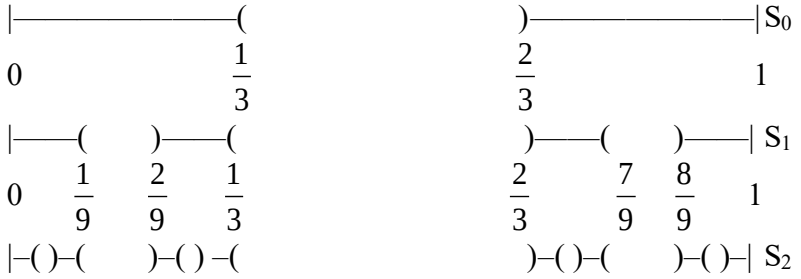
$a=$

(ii) 圖中的圓之圓心為 O，半徑為 1，A 和 B 是圓形上的點。
已知 $\dfrac{\text{陰影部分}}{\text{沒有陰影部分}}=\dfrac{\pi-2}{3\pi+2}$ 且 $\angle AOB=b^\circ$，求 b 的值。
The circle in the figure has centre O and radius 1, A and B are points on the circle.
Given that $\dfrac{\text{Area of shaded part}}{\text{Area of unshaded part}}=\dfrac{\pi-2}{3\pi+2}$
and $\angle AOB=b^\circ$, find the value of b.



$b=$

(iii) 圖形 S_0，S_1，S_2，…用以下方法構成：把綫段[0, 1]的中間三分之一取去，得到 S_0，把 S_0 的兩條組成綫段，每段的中間三分之一取去，得到 S_1，把 S_1 的四條組成綫段，每段的中間三分之一取去，得到 S_2，S_3、S_4 …等用類似方法獲得。求在構成 S_5 的過程中取去的綫段的總長度 c (答案以分數表示)。
A sequence of figures S_0, S_1, S_2, … are constructed as follows. S_0 is obtained by removing the middle third of [0,1] interval; S_1 by removing the middle third of each of the two intervals in S_0; S_2 by removing the middle third of each of the four intervals in S_1; S_3, S_4, … are obtained similarly. Find the total length c of the intervals removed in the construction of S_5 (Give your answer in fraction).



$c=$

(iv) 把所有整數用下表的方法編碼。若編碼 101 至 200 的所有整數之和為 d，求 d 的值。
All integers are coded as shown in the following table. If the sum of all integers coded from 101 to 200 is d, find the value of d.

$d=$

整數 Integer	…	…	–3	–2	–1	0	1	2	3	…	…
編碼 Code	…	…	7	5	3	1	2	4	6	…	…

FOR OFFICIAL USE

Score for accuracy ____ × Mult. factor for speed ____ = ____ Team No. ____

\+ Bonus score ____ Time ____ ____

Total score ____ Min. Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event 5 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $1\times2\times3 + 2\times3\times4 + 3\times4\times5 + \ldots + 10\times11\times12 = a$，求 a 的值。
If $1\times2\times3 + 2\times3\times4 + 3\times4\times5 + \ldots + 10\times11\times12 = a$, find the value of a.

$a =$

(ii) 已知 $5^x + 5^{-x} = 3$。若 $5^{3x} + 5^{-3x} = b$，求 b 的值。
Given that $5^x + 5^{-x} = 3$. If $5^{3x} + 5^{-3x} = b$, find the value of b.

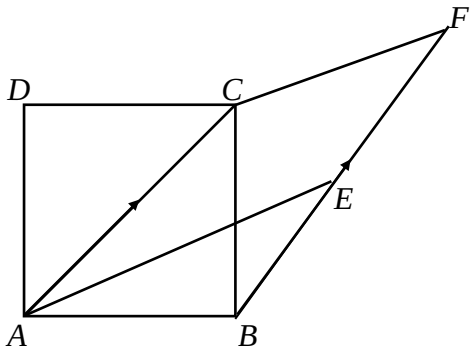
$b =$

(iii) 已知二次方程 $x^2 + mx + n = 0$ 的根為 98 和 99，且 $y = x^2 + mx + n$。
若 x 取 0、1、2、…、100，則有 c 個 y 的數值能被 6 整除。求 c 的值。
Given that the roots of equation $x^2 + mx + n = 0$ are 98 and 99 and $y = x^2 + mx + n$.
If x takes on the values of 0, 1, 2, …, 100, then there are c values of y that can be divisible by 6. Find the value of c.

$c =$

(iv) 在圖中，$ABCD$ 為一正方形，$BF \parallel AC$，且 $AEFC$ 為一菱形。
若 $\angle EAC = d°$，求 d 的值。
In the figure, $ABCD$ is a square, $BF \parallel AC$, and $AEFC$ is a rhombus.
If $\angle EAC = d°$, find the value of d.

$d =$



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Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

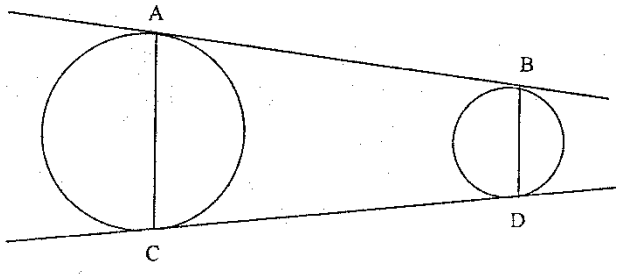
Min. Sec.

Hong Kong Mathematics Olympiad (1997-98)
Final Event Spare (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

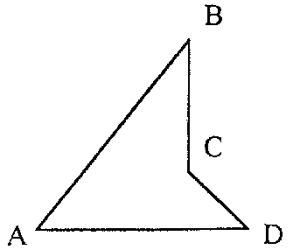
(i) 在圖中，有兩外公切綫，此外公切綫與圓相交於點 A、B、C 及 D。
若 $AC = 9$ cm，$BD = 3$ cm，$\angle BAC = 60°$及 $AB = s$ cm，求 s 的值。
In the figure, there are two common tangents. These common tangents meet the circles at points A, B, C and D. If $AC = 9$ cm, $BD = 3$ cm, $\angle BAC = 60°$ and $AB = s$ cm, find the value of s.

$s =$



(ii) 在圖中，$ABCD$ 為一四邊形，其中內角$\angle A$、$\angle B$ 及$\angle D$ 均為 45°。BC 的延綫與 AD 互相垂直。若 $AC = 10$，$BD = b$，求 b 的值。
In the figure, $ABCD$ is a quadrilateral, where the interior angles $\angle A$, $\angle B$ and $\angle D$ are all equal to 45°. When produced, BC is perpendicular to AD.
If $AC = 10$ and $BD = b$, find the value of b.

$b =$



(iii) 若 $\log_c 27 = 0.75$，求 c 的值。
If $\log_c 27 = 0.75$, find the value of c.

$c =$

(iv) 若數據 30，80，50，40，d 的平均數、眾數和中位數都相等，求 d 的值。
If the mean, mode and median of the data 30, 80, 50, 40, d are all equal, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy	×	Mult. factor for speed	=		Team No.	
				+ Bonus score	Time	Min. Sec.
				Total score		