Hong Kong Mathematics Olympiad 1997-1998
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。　　　　　　　　　時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 已知 $x^3 - 5x^2 + 2x + 8$ 能被 $(x - a)$ 和 $(x - 2a)$ 整除，且 a 為整數。求 a 的值。

    Given that $x^3 - 5x^2 + 2x + 8$ is divisible by $(x - a)$ and $(x - 2a)$, where a is an integer, find the value of a .

2. 已知 8、a、b 形成一等差級數，且 a、b、36 形成一等比級數。

    若 a 和 b 皆為正數，求 a、b 的和。

    Given that 8 , a , b form an A.P. and a, b, 36 form a G.P.

    If a and b are both positive numbers, find the sum of a and b .

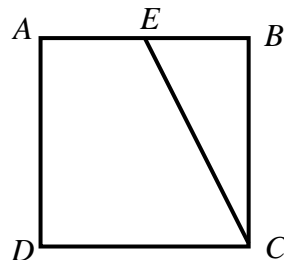3. 求下列方程式的最小實根：$\dfrac{x}{(x-4)(x+3)} = \dfrac{x}{(x+4)(x-6)}$ 。

    Find the smallest real root of the following equation: $\dfrac{x}{(x-4)(x+3)} = \dfrac{x}{(x+4)(x-6)}$.

4. 在圖一，$ABCD$ 為一正方形。E 為 AB 上的一點，使得 $BE = 1$ 及 $CE = 2$。求正方形 $ABCD$ 的面積。

    In figure 1, $ABCD$ is a square. E is a point on AB such that $BE = 1$ and $CE = 2$. Find the area of the square $ABCD$.

圖一
Figure 1

5. 若 $2x + 3 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$，求 x 的值。

    If $2x + 3 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$, find the value of x.

6. 已知 n 為一小於 1000 的正整數。若 n 能被 3 或 5 整除，求 n 之可能數值有多少個。

    Given that n is a positive integer which is less than 1000. If n is divisible by 3 or 5, find the number of possible values of n .

7.　在圖二，ABCD 為一長方形，其中 CD = 12，且 E 為 CD 上一點，使得 DE = 5。

若 M 為 AE 的中點，而 P、Q 為兩點分別躺於 AD 和 BC 上，使得 PMQ 為一直綫。

若 PM : MQ = 5 : k，求 k 的值。

In figure 2, ABCD is a rectangle with CD = 12. E is a point on CD such that DE = 5. M is the mid-point of AE and P, Q are points on AD and BC respectively such that PMQ is a straight line. If PM : MQ = 5 : k, find the value of k.
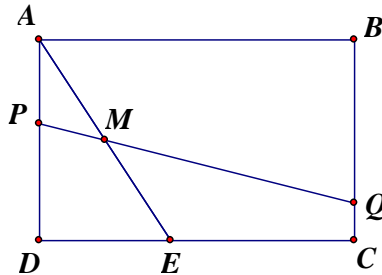
圖二

Figure 2

8.　求 $6^{20} - 5^{12} - 8$ 值的末位數字。

Find the last digit of the value of $6^{20} - 5^{12} - 8$.

9.　設 a 為方程 $\sqrt{\dfrac{x+2}{x-1}} + \sqrt{\dfrac{x-1}{x+2}} = \dfrac{5}{2}$ 的正根，求 a 的數值。

Let a be the positive root of the equation $\sqrt{\dfrac{x+2}{x-1}} + \sqrt{\dfrac{x-1}{x+2}} = \dfrac{5}{2}$, find the value of a .

10.　求 240 的所有正因數的和。

Find the sum of all positive factors of 240 .

*** 試卷完 End of Paper ***

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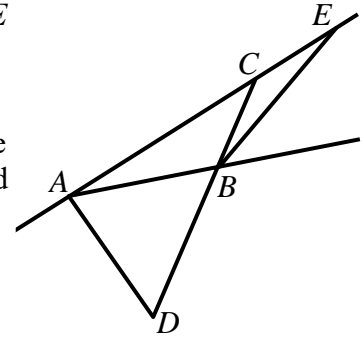
1.　若 $x + \dfrac{1}{x} = 2$ ，求 $x^3 + \dfrac{1}{x^3}$ 的值。

If $x + \dfrac{1}{x} = 2$, find the value of $x^3 + \dfrac{1}{x^3}$.

2.　在圖一，ABC 是一三角形，外角 A 和 B 的角平分綫 AD 和 BE 分別交 CB 和 AC 的延綫於 D 和 E。設 $AD = BE = AB$ 和 $\angle BAC = a°$，求 a 的值。

In Figure 1, ABC is a triangle. AD and BE are the bisectors of the exterior angles A and B respectively meeting CB and AC produced at D and E. Let $AD = BE = AB$ and $\angle BAC = a°$.
Find the value of a .

圖一　Figure 1

3.　若 $-6 \leq a \leq 4$ 及 $3 \leq b \leq 6$，求 $a^2 - b^2$ 的最大值。

If $-6 \leq a \leq 4$ and $3 \leq b \leq 6$, find the greatest value of $a^2 - b^2$.

4.　給出整數 a、b、c，使得 $a^2 = b^3 = c$。若 $c > 1$，求 c 的最小值。

Let a, b, c be integers such that $a^2 = b^3 = c$. If $c > 1$, find the smallest value of c .

5.　在圖二，平行四邊形 $ABCD$ 之面積為 120。點 M 和 N 分別為邊 AB 及 BC 之中點。AN 與 MD 及 BD 分別相交於點 P 及 Q。求 $BQPM$ 的面積。
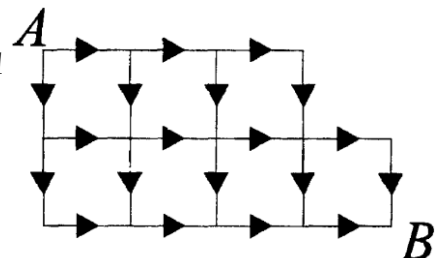
In figure 2, the area of the parallelogram $ABCD$ is 120.　M and N are the mid-points of AB and BC respectively.　AN intersects MD and BD at points P and Q respectively.　Find the area of $BQPM$.
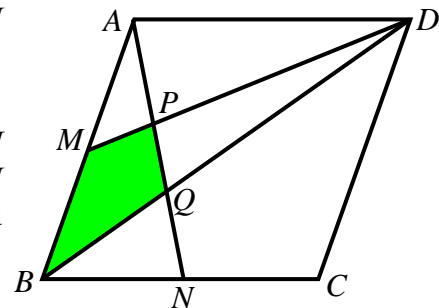
圖二　Figure 2

6.　在圖三，沿箭頭方向前進，求 A 到 B 點的不同路綫數目。

In figure 3, find the number of possible paths from point A to point B following the direction of arrow heads.
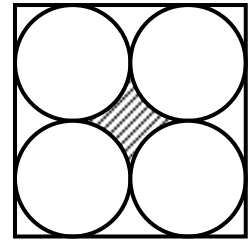

圖三　Figure 3

7.　求方程$(x - 2)(2x - 1) = 5$ 的最小實根。

Find the smallest real root of the equation $(x - 2)(2x - 1) = 5$.

8. 在圖四，四個半徑為 1 的圓緊緊地放在一個正方形內。求陰影部分的
    面積。(答案準確至最接近之整數)。
    In Figure 4, four circles with radius 1 touch each other inside a square.
    Find the shaded area. (Correct your answer to the nearest integer.)

圖四 Figure 4

9. 在圖五，*ABCD* 為正方形。點 *E*、*F*、*G*、*H* 分別為邊 *AB*、*BC*、*CD*、*DA* 之中點，
    求圖中直角三角形的數目。
    In figure 5, *ABCD* is a square and points *E*, *F*, *G*, *H* are the mid-points of sides *AB*, *BC*, *CD*,
    DA respectively, find the number of right-angled triangles in the figure.
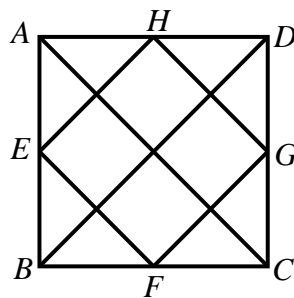
圖五 Figure 5

10. 某測驗共有 25 題多項選擇題。每題答對得 4 分，答錯扣 1 分。
    某學生全答所有題目，得分 70，問該生共答對多少題？
    A test is composed of 25 multiple-choice questions. 4 marks will be awarded for each correct
    answer and 1 mark will be deducted for each incorrect answer. A pupil answered all questions
    and got 70 marks. How many questions did the pupil answer correctly?

*** 試卷完 **End of Paper** ***