

Hong Kong Mathematics Olympiad 1998-1999
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。

時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 有一圓，其圓周是 14π cm。

若一弧所對的圓心角是 $\frac{1}{7}$ 個弧度，設這弧的長度是 X cm，求 X 的數值。

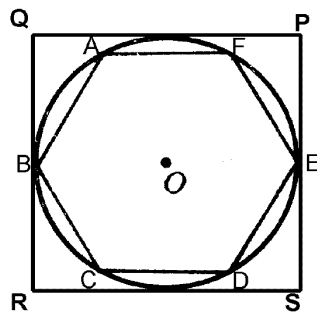
The circumference of a circle is 14π cm. Let X cm be the length of an arc of the circle, which subtends an angle of $\frac{1}{7}$ radian at the centre. Find the value of X .

2. 在圖一， $ABCDEF$ 是一正六邊形及其面積是 $3\sqrt{3}$ cm²。

設正方形 $PQRS$ 的面積是 X cm²，求 X 的值。

In Figure 1, $ABCDEF$ is a regular hexagon with area equal to $3\sqrt{3}$ cm².

Let X cm² be the area of the square PQRS, find the value of X .



圖一
Figure 1

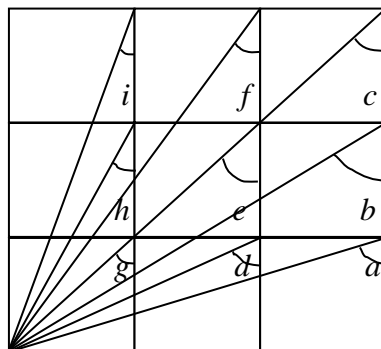
3. 已知 8 點，其中沒有任何 3 點是共線的。求以任意 3 點作為三角形頂點的三角形的個數。

8 points are given and no three of them are collinear.

Find the number of triangles formed by using any 3 of the given points as vertices.

4. 在圖二，有一個 3×3 正方形。設 $\angle a + \angle b + \dots + \angle i = X^\circ$ ，求 X 的數值。

In Figure 2, there is a 3×3 square. Let $\angle a + \angle b + \dots + \angle i = X^\circ$, find the value of X .



圖二 Figure 2

5. 在 0 至 10^6 之間，有多少個整數 n ，使得 n^3 的個位數字是 1?

How many integers n are there between 0 and 10^6 , such that the unit digit of n^3 is 1?

6. 已知 a 、 b 、 c 是正整數，且滿足 $a < b < c = 100$ ，

求以 a cm、 b cm、 c cm 為邊長的三角形的個數。

Given that a, b, c are positive integers and $a < b < c = 100$,

find the number of triangles formed with sides equal a cm, b cm and c cm.

7. 一班青年參加旅行，他們同意所有消費平均攤分。整個活動，他們共用去 288 元。其中有一位成員無法支付其所應付出的部份。其他成員願意各多付 4 元，湊夠其數。問共有多少青年參加這次旅行。

A group of youngsters went for a picnic. They agreed to share all expenses. The total amount used was \$288. One youngster had no money to pay his share, and each of the others had to pay \$4 more to cover the expenses. How many youngsters were there in the group?

8. 某兩位數其值等於它的位值的和的 4 倍。若將該數的個位和十位數字相調，這個新兩位數的值比其位值的和的 5 倍多出 18。求該數。

A two-digit number is equal to 4 times the sum of the digits, and the number formed by reversing the digits exceeds 5 times the sum of the digits by 18. What is the number?

9. 已知下列序列的第 1001 項的分母為 46，求該項的分子。

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$$

Given that the denominator of the 1001th term of the following sequence is 46, find the numerator of this term.

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \dots$$

10. 下列加法算式中，若字母‘S’代表 4，那麼字母‘A’代表甚麼數字？

In the following addition, if the letter ‘S’ represents 4, what digit does the letter ‘A’ represent?

$$\begin{array}{r} \text{SEE} \\ \text{SEE} \\ \text{SEE} \\ + \text{YES} \\ \hline \text{EASY} \end{array}$$

*** 試卷完 End of Paper ***

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1. 若 a 是質數，且滿足 $a^2 - 2a - 15 < 0$ ，求 a 的最大值。

If a is a prime number and $a^2 - 2a - 15 < 0$, find the greatest value of a .

2. 若 $a : b : c = 3 : 4 : 5$ 及 $a + b + c = 48$ ，求 $a - b - c$ 的值。

If $a : b : c = 3 : 4 : 5$ and $a + b + c = 48$, find the value of $a - b - c$.

3. 求 $\log(\sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}})$ 的值。

Find the value of $\log(\sqrt{3+\sqrt{5}} + \sqrt{3-\sqrt{5}})$.

4. 求直線 $x + 4y - 2 = 0$ 與兩條坐標軸所圍成的三角形的面積。

Find the area enclosed by the straight line $x + 4y - 2 = 0$ and the two coordinate axes.

5. 把由 1 開始的自然數依次寫下去，直寫到第 198 位為止 123456789101112.....。

198 位

求所得數被 9 除的餘數。

Natural numbers are written in order starting from 1 until 198th digit as shown 123456789101112...... If the number obtained is divided by 9, find the remainder.

198 digits

6. $2, a, 5, b, 8$ 的平均數為 6。若 n 為 $a, 2a+1, 11, b, 2b+3$ 的平均數，求 n 的值。

The average of $2, a, 5, b, 8$ is 6. If n is the average of $a, 2a+1, 11, b, 2b+3$, find the value of n .

7. 若 $p = 2x^2 - 4xy + 5y^2 - 12y + 16$ ，其中 x 及 y 皆為實數，求 p 的最小值。

If $p = 2x^2 - 4xy + 5y^2 - 12y + 16$, where x and y are real numbers, find the least value of p .

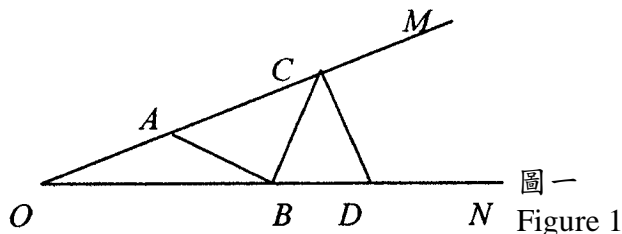
8. 求 333^{335} 的個位數字。

Find the units digit of 333^{335} .

9. 在圖一， $\angle MON = 20^\circ$ ， A 為 OM 上的一點， $OA = 4\sqrt{3}$ ， D 為 ON 上的一點， $OD = 8\sqrt{3}$ ， C 為 AM 上的任意一點， B 為 OD 上的任意一點。

若 $\ell = AB + BC + CD$ ，求 ℓ 的最小值。

In Figure 1, $\angle MON = 20^\circ$, A is a point on OM , $OA = 4\sqrt{3}$, D is a point on ON , $OD = 8\sqrt{3}$, C is any point on AM , B is any point on OD . If $\ell = AB + BC + CD$, find the least value of ℓ .



圖一

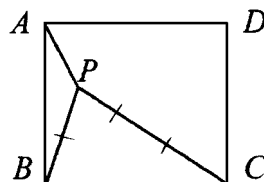
Figure 1

10. 在圖二， P 為正方形 $ABCD$ 內一點， $PA = a$ ， $PB = 2a$ ， $PC = 3a$ ($a > 0$)。

若 $\angle APB = x^\circ$ ，求 x 的值。

In figure 2, P is a point inside the square $ABCD$, $PA = a$, $PB = 2a$, $PC = 3a$ ($a > 0$).

If $\angle APB = x^\circ$, find the value of x .



圖二

Figure 2

*** 試卷完 End of Paper ***