Hong Kong Mathematics Olympiad 1999-2000
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 設 $x = 0.\dot{1}\dot{7} + 0.0\dot{1}\dot{7} + 0.00\dot{1}\dot{7} + \ldots$，求 x 的值。

   Let $x = 0.\dot{1}\dot{7} + 0.0\dot{1}\dot{7} + 0.00\dot{1}\dot{7} + \ldots$, find the value of x.

2. 解下列方程：

   $$\frac{1}{x+12} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \cdots + \frac{1}{(x+10)(x+11)} + \frac{1}{(x+11)(x+12)} = \frac{1}{4}。$$

   Solve the following equation:

   $$\frac{1}{x+12} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \cdots + \frac{1}{(x+10)(x+11)} + \frac{1}{(x+11)(x+12)} = \frac{1}{4}.$$

3. 用數字 0、 1、 2、 5 可以組成多少個能被 5 整除的三位數？(若數字不可以重複使用。)
   Using digits 0, 1, 2, and 5, how many 3-digit numbers can be formed, which are divisible by 5? (If no digit may be repeated.)

4. 在圖一，有一個 4×3 的矩形蜘蛛網。若有一隻蜘蛛沿著網絲爬行。而其爬行方向衹可向東或向北。該蜘蛛由 A 點到 C 點共有多少種可能路徑？
   Figure 1 represents a 4×3 rectangular spiderweb. If a spider walks along the web from A to C and it always walks either due East or due North. Find the total number of possible paths.

Figure 1 圖一

5. 在圖二，設 $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G = x°$，求 x 的值。
   In Figure 2, let $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G = x°$, find the value of x.

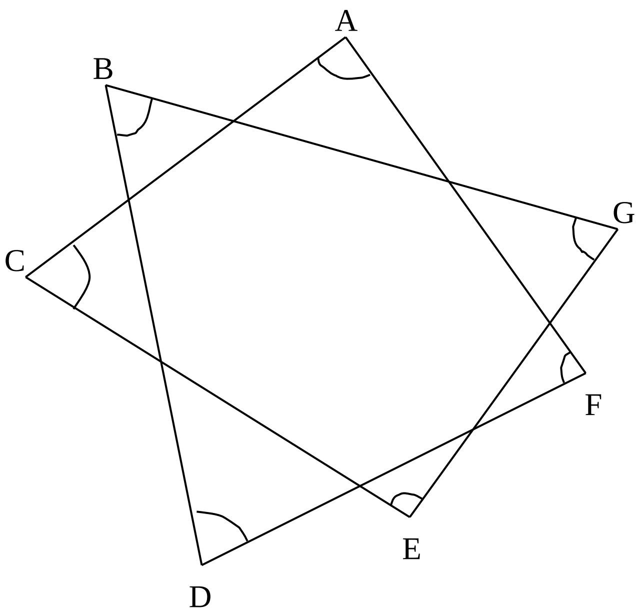

Figure 2 圖二

6. 於一白紙上，畫有 20 條直綫。該 20 條直綫，並沒有兩條或兩條以上是平行的，也沒有三條或三條以上的直綫共點，問這 20 條直綫最多可構成多少個交點？
Twenty straight lines were drawn on a white paper. Among them, no two or more straight lines are parallel; also no three or more than three straight lines are concurrent. What is the maximum number of intersections that these 20 lines can form?

7. 某一家庭有兩個孩子，已知其中一個孩子是女的，求該家庭的另一個孩子亦是女兒的概率是多少？(假設生男、生女的概率相等。)
In a family of 2 children, given that one of them is a girl, what is the probability of having another girl? (Assuming equal probabilities of boys and girls.)

8. 有一個六位數，其個位數字為「1」，若將該個位數字「1」移至十萬位，其原來的十萬位數字、萬位數字、千位數字、⋯ 皆向右順移一個位。新的六位數的值為原來的六位數的值的 $\frac{1}{3}$，求原來的六位數。
A particular 6-digit number has a unit-digit “1”. Suppose this unit-digit “1” is moved to the place of hundred thousands, while the original ten thousand-digit, thousand-digit, hundred-digit, ⋯ are moved one digit place to the right. The value of the new 6-digit number is one-third of the value of the original 6-digit number. Find the original 6-digit number.

9. 求 $\frac{12\sin^2 48^\circ + 12\sin^2 42^\circ}{\sin 330^\circ \tan 135^\circ - \sin^2 48^\circ \sin^2 42^\circ \tan 180^\circ}$ 的值。
Find the value of $\frac{12\sin^2 48^\circ + 12\sin^2 42^\circ}{\sin 330^\circ \tan 135^\circ - \sin^2 48^\circ \sin^2 42^\circ \tan 180^\circ}$.

10. 求直綫 $3x - y - 4 = 0$ 與點 $(2, 2)$ 的最短距離。
Find the shortest distance between the line $3x - y - 4 = 0$ and the point $(2, 2)$.

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1. 如果 a 是 $x^2+2x+3=0$ 的根，求 $\dfrac{a^5+3a^4+3a^3-a^2}{a^2+3}$ 的值。
   If a is a root of $x^2+2x+3=0$, find the value of $\dfrac{a^5+3a^4+3a^3-a^2}{a^2+3}$.

2. 方程 $(\cos^2\theta-1)(2\cos^2\theta-1)=0$ 恰有 n 個根，其中 $0^\circ<\theta<360^\circ$。求 n 的值。
   There are exactly n roots in the equation $(\cos^2\theta-1)(2\cos^2\theta-1)=0$,
   where $0^\circ<\theta<360^\circ$. Find the value of n.

3. 求 2004^{4006} 的個位數。
   Find the units digit of 2004^{2006}.

4. 設 $x=|y-m|+|y-10|+|y-m-10|$，其中 $0<m<10$ 和 $m\le y\le 10$。 求 x 的最小值。
   Let $x=|y-m|+|y-10|+|y-m-10|$, where $0<m<10$ and $m\le y\le 10$.
   Find the minimum value of x.

5. 有 5 個分別標上 A、B、C、D、E 的球及 5 個分別標上 A、B、C、D、E 的袋，每個袋放一個球。求恰好有 3 個球的標號與袋的標號相同的投放方法總數。
   There are 5 balls with labels A, B, C, D, E respectively and there are 5 pockets with labels A, B, C, D, E respectively. A ball is put into each pocket. Find the number of ways in which exactly 3 balls have labels that match the labels on the pockets.

6. 如圖一，ΔPQR 為一等邊三角形，$PT = RS$；PS、QT 相交於 M；QN 垂直 PS 於 N。設 $\angle QMN = x^\circ$，求 x 的值。
   In Figure 1, ΔPQR is an equilateral triangle, $PT = RS$; PS, QT meet at M; and QN is perpendicular to PS at N. Let $\angle QMN = x^\circ$, find the value of x .

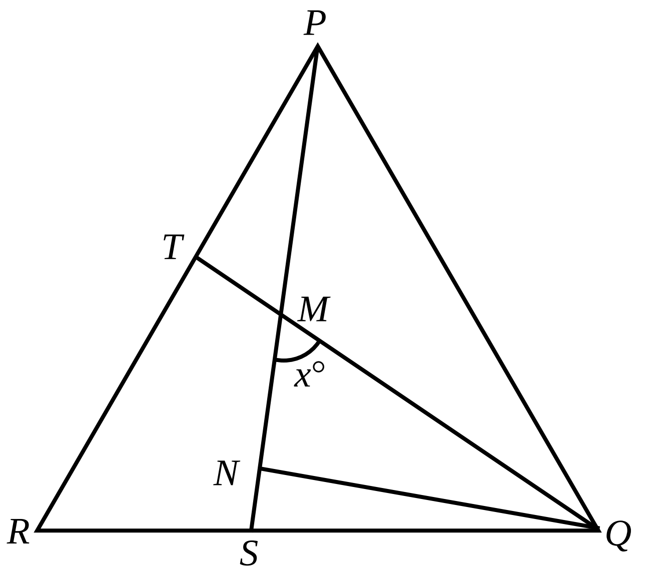

Figure 1（圖一）

7. 如圖二，已知三等圓互相外切，且內切於矩形 $PQRS$，求 $\frac{QR}{SR}$ 的值。

(取 $\sqrt{3}=1.7$ 及答案須準確至二個小數位)

In Figure 2, three equal circles are tangent to each other, and inscribed in rectangle *PQRS*, find the value of $\frac{QR}{SR}$. (Use $\sqrt{3}=1.7$ and give the answer correct to 2 decimal places)

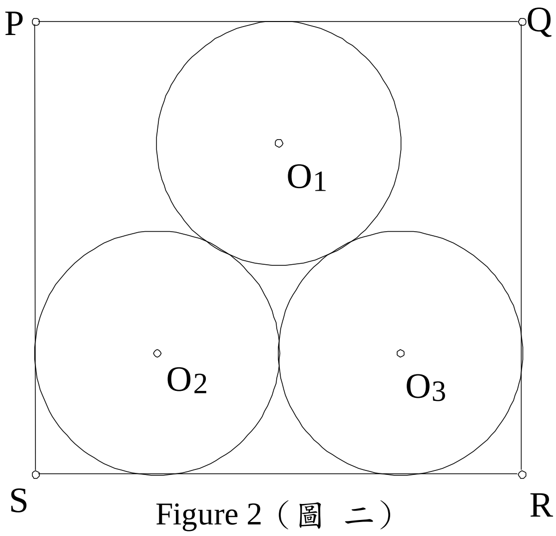

Figure 2（圖 二）

8. 兩個正整數之和為 29，求此兩數平方和的最小值。

The sum of two positive integers is 29, find the minimum value of the sum of their squares.

9. 設 $x=\sqrt{3+\sqrt{3}}$ 及 $y=\sqrt{3-\sqrt{3}}$，求 $x^2(1+y^2)+y^2$ 的值。

Let $x=\sqrt{3+\sqrt{3}}$ and $y=\sqrt{3-\sqrt{3}}$, find the value of $x^2(1+y^2)+y^2$.

10. 袋內有球 9 個，分別標上整數 1 到 9。甲從袋中隨機地抽出一個球並把它放回，乙再從同一袋中隨機地抽出一個球。把兩球上的整數相加，設 n 為該和的個位數字，P(n) 為 n 出現的概率。求 n 的值使得 P(n)為最大。

There are nine balls in a pocket, each one having an integer label from 1 to 9. *A* draws a ball randomly from the pocket and puts it back, then *B* draws a ball randomly from the same pocket. Let n be the unit digit of the sum of numbers on the two balls drawn by *A* and *B*, and P(n) be the probability of the occurrence of n. Find the value of n such that P(n) is the maximum.

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