

Hong Kong Mathematics Olympiad 2001-2002
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。

時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 袋中有數字卡 9 張，其數字分別為 1 至 9。若隨機一次抽出 3 張，求被抽出的卡的數字全是奇數的概率。(答案以分數表達，並化至最簡。)

There are 9 cards, numbered from 1 to 9, in a bag. If 3 cards are drawn together at random, find the probability that all are odd. (Express your answer in the simplest fraction.)

2. 已知 $a^3 = 150b$ ，且 a 和 b 都是正整數。求 b 的最小值。

Given $a^3 = 150b$, and a, b are positive integers, find the least value of b .

3. 已知 $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$ ，且 a, b 是自然數。若 $a + b = y$ ，求 y 的值。

Suppose $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$, and a, b are natural numbers.

If $a + b = y$, find the value of y .

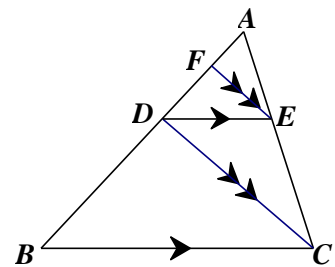
4. 把數字 2, 3, 4, 5 組成沒有重複數字的四位數，求這些四位數的和。

Each of the digits 2, 3, 4, 5 can be used once and only once in writing a four-digit number. Find the sum of all such numbers.

5. 在 $\triangle ABC$ ， $DE \parallel BC$ ， $FE \parallel DC$ ， $AF = 2$ ， $FD = 3$ 和 $DB = X$ 。
求 X 的值。

In $\triangle ABC$, $DE \parallel BC$, $FE \parallel DC$, $AF = 2$, $FD = 3$ and $DB = X$.

Find the value of X .



6. 若一圓內接四邊形的四邊長度為 9, 10, 10 和 21，求該圓內接四邊形的面積。

If the lengths of the sides of a cyclic quadrilateral are 9, 10, 10 and 21 respectively, find the area of the cyclic quadrilateral.

7. 若 $\frac{(a-b)(c-d)}{(b-c)(d-a)} = 3$ ，求 $\frac{(a-c)(b-d)}{(a-b)(c-d)}$ 的值。

If $\frac{(a-b)(c-d)}{(b-c)(d-a)} = 3$, find the value of $\frac{(a-c)(b-d)}{(a-b)(c-d)}$.

8. 若 $x^3 + kx^2 + 3$ 除以 $x + 3$ ，其餘數較被 $x + 1$ 除所得的餘數少 2。求 k 的值。

When the expression $x^3 + kx^2 + 3$ is divided by $x + 3$, the remainder is 2 less than when divided by $(x + 1)$. Find the value of k .

9. 已知圓形上的某扇形的周界為 18。當圓的半徑為 r 時，該扇形的面積達至最大值，求 r 的值。

Given that the perimeter of a sector of a circle is 18. When the radius is r , the area of the sector attains the maximum value, find the value of r .

10. 已知 $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ ，求 $f(5)$ 的值。

Given $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, find the value of $f(5)$.

*** 試卷完 End of Paper ***

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Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。

時限：20 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 有糖果一袋分配給甲、乙、丙三人。甲、乙、丙三人依次所得的糖果數目的比是 5 : 4 : 3。若把糖果重新分配給甲、乙、丙三人使其比依次為 7 : 6 : 5，則其中一人比原本所得的數目多了 40 粒，問此人原本所得的糖果數目。

A bag of sweets is distributed to three persons A, B and C . The numbers of sweets obtained by A, B and C are in the ratios of 5 : 4 : 3 respectively. If the sweets are re-distributed to A, B, C according to the ratios 7 : 6 : 5 respectively, then one of them would get 40 more sweets than his original number. Find the original number of sweets obtained by this person.

2. 已知 a, b, c 為三個連續奇數且 $b^3 = 3375$ ，求 ac 的數值。

Given that a, b, c are three consecutive odd numbers and $b^3 = 3375$, find the value of ac .

3. 設在直角坐標平面上不等式 $|x| + |y| \leq 3$ 圍出的多邊形內面積為 p ，求 p 的數值。

Let p be the area of the polygon formed by the inequality $|x| + |y| \leq 3$ in the Cartesian plane. Find the value of p .

4. 求 $7^{2003} \div 100$ 的餘數。

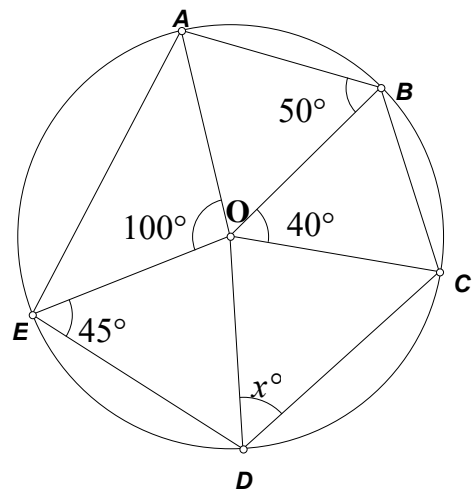
Find the remainder of $7^{2003} \div 100$.

5. 如果實數 x, y 滿足方程 $x^2 + y^2 + 3xy = 35$ ，求 xy 的最大值。

If real numbers x, y satisfy the equation $x^2 + y^2 + 3xy = 35$, find the maximum value of xy .

6. 如圖一，點 A, B, C, D, E 位於以 O 為圓心的一個圓上。已知 $\angle DEO = 45^\circ$ ， $\angle AOE = 100^\circ$ ， $\angle ABO = 50^\circ$ ， $\angle BOC = 40^\circ$ 及 $\angle ODC = x^\circ$ ，求 x 的數值。

In figure 1, points A, B, C, D, E are on a circle with centre at O . Given $\angle DEO = 45^\circ$, $\angle AOE = 100^\circ$, $\angle ABO = 50^\circ$, $\angle BOC = 40^\circ$, and $\angle ODC = x^\circ$, find the value of x .



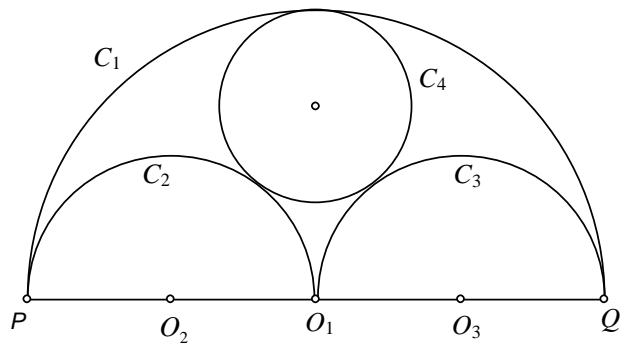
圖一(Figure 1)

7. 將 20 個球放入 2 個袋中，每袋 10 個球，每袋的球分別標上數字 1 到 10，其中一個袋的球全為白色，另一個袋的球全為黑色。若從兩個袋中任意各取一個球，求白球上的數字較黑球上的數字為大的概率。

20 balls are put into 2 bags with 10 balls in each bag. The balls in each bag are labeled numbers 1 to 10, all balls in one bag are white and all balls in the other bag are black. If one ball is drawn from each of two bags, find the probability that the number of the white ball is greater than that of the black ball.

8. 如圖二， PQ 、 PO_1 、 O_1Q 分別是以 O_1 、 O_2 、 O_3 為圓心的半圓 C_1 、 C_2 、 C_3 的直徑，圓 C_4 內切於半圓 C_1 及外切於半圓 C_2 、 C_3 。若 $PQ = 24$ ，求圓 C_4 的面積 (取 $\pi = 3$)。

In figure 2, PQ , PO_1 , O_1Q are diameters of semi-circles C_1 , C_2 , C_3 with centres at O_1 , O_2 , O_3 respectively, and the circle C_4 touches C_1 , C_2 , and C_3 . If $PQ = 24$, find the area of circle C_4 . (Take $\pi = 3$).



圖二 (Figure 2)

9. 已知正整數 a 、 b 滿足方程 $ab - a - b = 12$ ，求 ab 的值。
Given that a and b are positive integers satisfying the equation $ab - a - b = 12$, find the value of ab .
10. 已知三角形 ABC 中的 $\angle A$ 為一直角， $\sin^2 C - \cos^2 C = \frac{1}{4}$ ， $AB = \sqrt{40}$ 及 $BC = x$ ，求 x 的數值。
Given that $\angle A$ is a right angle in triangle ABC , $\sin^2 C - \cos^2 C = \frac{1}{4}$, $AB = \sqrt{40}$ and $BC = x$, find the value of x .