Hong Kong Mathematics Olympiad (2005 – 2006)

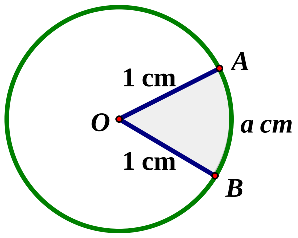
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 a 為實數且滿足方程 $\log_2(x+3)-\log_2(x+1)=1$，求 a 的值。
If a is a real number satisfying $\log_2(x+3)-\log_2(x+1)=1$, find the value of a.

$a =$

2. 如圖一，O 是半徑 1 cm 的圓的圓心。若弧 AB 的長度是 a cm 及著色部份扇形 OAB 的面積是 b cm^2，求 b 的值。(取 $\pi=3$)
In Figure 1, O is the centre of the circle with radius 1 cm. If the length of the arc AB is equal to a cm and the area of the shaded sector OAB is equal to b cm^2, find the value of b. (Take $\pi=3$)



圖一 Figure 1

$b =$

3. 一個正 C 邊形的一隻內角是 $288b°$，求 C 的值。
An interior angle of a regular C-sided polygon is $288b°$, find the value of C.

$C =$

4. 已知 10 是方程 $kx^2+2x+5=0$ 的一個根，其中 k 為常數。若 D 是另一個根，求 D 的值。
Given that C is a root of the equation $kx^2+2x+5=0$, where k is a constant. If D is another root, find the value of D.

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $a:b:c=6:3:1$。若 $R=\dfrac{3b^2}{2a^2+bc}$，求 R 的值。

Given that $a:b:c=6:3:1$. If $R=\dfrac{3b^2}{2a^2+bc}$, find the value of R .

$R=$

2. 已知 $\dfrac{|k+R|}{|R|}=0$，若 $S=\dfrac{|k+2R|}{|2k+R|}$，求 S 的值。

Given that $\dfrac{|k+R|}{|R|}=0$. If $S=\dfrac{|k+2R|}{|2k+R|}$, find the value of S .

$S=$

3. 已知 $T=\sin 50°\times(S+\sqrt{3}\times\tan 10°)$，求 T 的值。

Given that $T=\sin 50°\times(S+\sqrt{3}\times\tan 10°)$, find the value of T .

$T=$

4. 已知 x_0 和 y_0 是實數且滿足方程組 $\begin{cases} y=\dfrac{T}{x} \\ y=|x|+T \end{cases}$，

若 $W=x_0+y_0$，求 W 的值。

Given that x_0 and y_0 are real numbers satisfying the system of equations $\begin{cases} y=\dfrac{T}{x} \\ y=|x|+T \end{cases}$.

If $W=x_0+y_0$, find the value of W .

$W=$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $\frac{2x-3}{x^2-x}=\frac{A}{x-1}+\frac{B}{x}$，其中 A 和 B 是常數。若 $S=A^2+B^2$，求 S 的值。

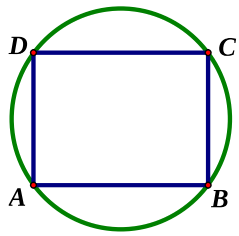
   Given that $\frac{2x-3}{x^2-x}=\frac{A}{x-1}+\frac{B}{x}$, where A and B are constants.

   If $S=A^2+B^2$, find the value of S.

   $S=$

2. 如圖一，$ABCD$ 是圓內長方形，$AB=(S-2)$ cm 及 $AD=(S-4)$ cm。若圓形的圓周是 R cm，求 R 的值。(取 $\pi=3$)

   In Figure 1, $ABCD$ is an inscribed rectangle, $AB=(S-2)$ cm and $AD=(S-4)$ cm. If the circumference of the circle is R cm, find the value of R. (Take $\pi=3$)



圖一 Figure 1

   $R=$

3. 已知整數 x 和 y 滿足 $\frac{R}{2}xy=21x+20y-13$。若 $T=xy$，求 T 的值。

   Given that x and y are integers satisfying the equation $\frac{R}{2}xy=21x+20y-13$.

   If $T=xy$, find the value of T.

   $T=$

4. 設 a 是方程 $x^2-2x-T=0$ 的一個正根。若 $P=3+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{a}}}}$，求 P 的值。

   Let a be the positive root of the equation $x^2-2x-T=0$.

   If $P=3+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{a}}}}$, find the value of P.

   $P=$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

\+ Bonus score [ ]

Time [ ] Min. [ ] Sec.

Total score [ ]

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $\frac{k}{4}=\left(1+\frac{1}{2}+\frac{1}{3}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\times\left(\frac{1}{2}+\frac{1}{3}\right)$，求 k 的值。
Let $\frac{k}{4}=\left(1+\frac{1}{2}+\frac{1}{3}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\times\left(\frac{1}{2}+\frac{1}{3}\right)$, find the value of k.

$k =$

2. 設 x 和 y 是實數且滿足方程 $y^2+4y+4+\sqrt{x+y+k}=0$。若 $r=|xy|$，求 r 的值。
Let x and y be real numbers satisfying the equation $y^2+4y+4+\sqrt{x+y+k}=0$.
If $r=|xy|$, find the value of r.

$r =$

3. 如圖一，八個正數排成一列，從第三個數開始，每個數都等於前面兩個數的乘積。已知第五個是 $\frac{1}{r}$，而第八個數是 $\frac{1}{r^4}$。若第一個是 s，求 s 的值。
In Figure 1, there are eight positive numbers in series. Starting from the 3rd number, each number is the product of the previous two numbers. Given that the 5th number is $\frac{1}{r}$ and the 8th number is $\frac{1}{r^4}$.
If the first number is s, find the value of s.

$s =$

s				$\frac{1}{r}$			$\frac{1}{r^4}$

圖一 Figure 1

4. 設 $[x]$ 表示不大於 x 的最大整數，例如 $[2.5]=2$。
若 $w=1+[10\times s^2]+[10\times s^4]+[10\times s^6]+\ldots+[10\times s^{2n}]+\cdots$，求 w 的值。
Let $[x]$ be the largest integer not greater than x. For example, $[2.5]=2$.
Let $w=1+[10\times s^2]+[10\times s^4]+[10\times s^6]+\ldots+[10\times s^{2n}]+\cdots$, find the value of w.

$w =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ] Team No. [ ]

\+ Bonus score [ ] Time [ ] [ ]

Total score [ ] Min. Sec.

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 k 為實數。若 $x^2+2kx-3k^2$ 能被 $x-1$ 整除，求 k 最大可能的值。
Given that k is a real number. If $x^2+2kx-3k^2$ can be divisible by $x-1$, find the greatest value of k.

$k =$

2. 已知 $x=x_0$ 及 $y=y_0$ 滿足方程組 $\begin{cases}\frac{x}{3}+\frac{y}{5}=1\\ \frac{x}{5}+\frac{y}{3}=1\end{cases}$。若 $B=\frac{1}{x_0}+\frac{1}{y_0}$，求 B 的值。

Given that $x=x_0$ and $y=y_0$ satisfy the system of equations $\begin{cases}\frac{x}{3}+\frac{y}{5}=1\\ \frac{x}{5}+\frac{y}{3}=1\end{cases}$.

If $B=\frac{1}{x_0}+\frac{1}{y_0}$, find the value of B.

$B =$

3. 已知 $x=2+\sqrt{3}$ 是方程 $x^2-(\tan\alpha+\cot\alpha)x+1=0$ 的一個根。
若 $C=\sin\alpha\times\cos\alpha$，求 C 的值。
Given that $x=2+\sqrt{3}$ is a root of the equation $x^2-(\tan\alpha+\cot\alpha)x+1=0$.
If $C=\sin\alpha\times\cos\alpha$, find the value of C.

$C =$

4. 設 a 為整數。若不等式 $|x+1|<a-1.5$ 沒有整數解，求 a 最大可能的值。
Let a be an integer. If the inequality $|x+1|<a-1.5$ has no integral solution, find the greatest value of a.

$a =$

FOR OFFICIAL USE

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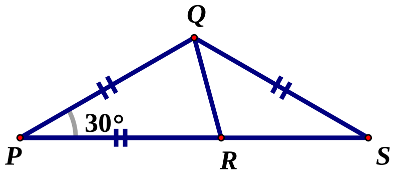
			+	Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一， PRS 是一直綫， $PQ = PR = QS$ 及 $\angle QPR = 30°$。若 $\angle RQS = w°$，求 w 的值。
In Figure 1, PRS is a straight line, $PQ = PR = QS$ and $\angle QPR = 30°$. If $\angle RQS = w°$, find the value of w.



圖一 Figure 1

$w =$

2. 設 $f(x) = px^7 + qx^3 + rx - 5$，其中 p、q 及 r 是實數。
若 $f(-6) = 3$ 及 $z = f(6)$，求 z 的值。
Let $f(x) = px^7 + qx^3 + rx - 5$, where p, q and r are real numbers.
If $f(-6) = 3$ and $z = f(6)$, find the value of z.

$z =$

3. 若 $n \neq 0$ 及 $s = \left(\frac{20}{2^{2n+4} + 2^{2n+2}}\right)^{\frac{1}{n}}$，求 s 的值。

If $n \neq 0$ and $s = \left(\frac{20}{2^{2n+4} + 2^{2n+2}}\right)^{\frac{1}{n}}$, find the value of s.

$s =$

4. 已知 x 和 y 是正整數及 $x + y + xy = 54$。若 $t = x + y$，求 t 的值。
Given that x and y are positive integers and $x + y + xy = 54$.
If $t = x + y$, find the value of t.

$t =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $r=2006\times\dfrac{\sqrt{8}-\sqrt{2}}{\sqrt{2}}$，求 r 的值。
Given that $r=2006\times\dfrac{\sqrt{8}-\sqrt{2}}{\sqrt{2}}$, find the value of r.

$r=$

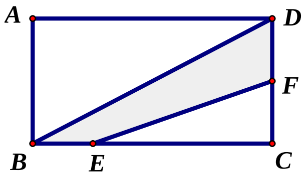
2. 已知 $6^{x+y}=36$ 及 $6^{x+5y}=216$，求 x 的值。
Given that $6^{x+y}=36$ and $6^{x+5y}=216$, find the value of x.

$x=$

3. 已知 $\tan x+\tan y+1=\cot x+\cot y=6$。若 $z=\tan(x+y)$，求 z 的值。
Given that $\tan x+\tan y+1=\cot x+\cot y=6$.
If $z=\tan(x+y)$, find the value of z.

$z=$

4. 如圖一，$ABCD$ 是一長方形，F 是 CD 的中點及 $BE:EC=1:3$。若長方形 $ABCD$ 的面積是 12 cm^2 及陰影部份 $BEFD$ 的面積是 $R\text{ cm}^2$，求 R 的值。
In Figure 1, $ABCD$ is a rectangle, F is the midpoint of CD and $BE:EC=1:3$. If the area of the rectangle $ABCD$ is 12 cm^2 and the area of $BEFD$ is $R\text{ cm}^2$, find the value of R.



圖一 Figure 1

$R=$

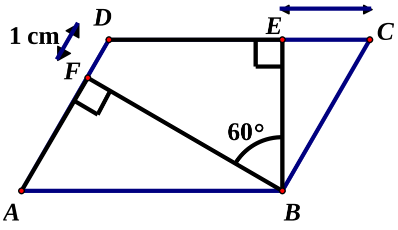
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，平行四邊形 $ABCD$ 中，$BE \perp CD$，$BF \perp AD$，$CE = 2$ cm，$DF = 1$ cm 及 $\angle EBF = 60°$。若平行四邊形 $ABCD$ 的面積是 R cm^2，求 R 的值。
In Figure 1, $ABCD$ is a parallelogram, $BE \perp CD$, $BF \perp AD$, $CE = 2$ cm, $DF = 1$ cm and $\angle EBF = 60°$. If the area of the parallelogram $ABCD$ is R cm^2, find the value of R.



圖一 Figure 1

$R =$

2. 已知 a 和 b 是正數且 $a + b = 2$。若 $S = \left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2$，求 S 的最小值。
Given that a and b are positive numbers and $a + b = 2$.
If $S = \left(a + \frac{1}{a}\right)^2 + \left(b + \frac{1}{b}\right)^2$, find the minimum value S.

$S =$

3. 設 $2^x = 7^y = 196$。若 $T = \frac{1}{x} + \frac{1}{y}$，求 T 的值。
Let $2^x = 7^y = 196$. If $T = \frac{1}{x} + \frac{1}{y}$, find the value of T.

$T =$

4. 若 $W = 2006^2 - 2005^2 + 2004^2 - 2003^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$，求 W 的值。
If $W = 2006^2 - 2005^2 + 2004^2 - 2003^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2$, find the value of W.

$W =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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