Hong Kong Mathematics Olympiad 2005-2006
Heat Event (Individual)

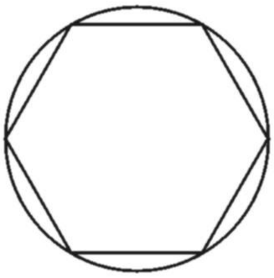
除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 設 $\sqrt{20+\sqrt{300}}=\sqrt{x}+\sqrt{y}$ 及 $w=x^2+y^2$，求 w 的值。

   Let $\sqrt{20+\sqrt{300}}=\sqrt{x}+\sqrt{y}$ and $w=x^2+y^2$, find the value of w.

2. 如圖一，一個正六邊形內接於一個圓周為 4 m 的圓內。設該正六邊形的面積是 $A\text{ m}^2$，求 A 的值。(取 $\pi=\frac{22}{7}$)

   In Figure 1, a regular hexagon is inscribed in a circle with circumference 4 m. If the area of the regular hexagon is $A\text{ m}^2$, find the value of A. (Take $\pi=\frac{22}{7}$)



圖一

Figure 1

3. 已知 $\dfrac{1}{2+\dfrac{3}{1+\dfrac{1}{x}}}=\dfrac{5}{28}$，求 x 的值。

   Given that $\dfrac{1}{2+\dfrac{3}{1+\dfrac{1}{x}}}=\dfrac{5}{28}$, find the value of x.

4. 設 $A=\dfrac{2006}{20052005^2-20052004\times 20052006}$，求 A 的值。

   Let $A=\dfrac{2006}{20052005^2-20052004\times 20052006}$, find the value of A.

5. 已知 $4\sec^2\theta°-\tan^2\theta°-7\sec\theta°+1=0$ 及 $0°\le\theta°\le 180°$，求 θ 的值。

   Given that $4\sec^2\theta°-\tan^2\theta°-7\sec\theta°+1=0$ and $0°\le\theta°\le 180°$, find the value of θ.

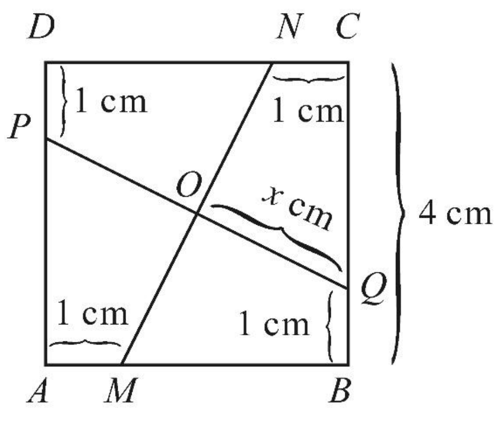
6. 已知 w、x、y 和 z 是正整數且滿足方程 $w+x+y+z=12$。若方程有 W 組不同的正整數解，求 W 的值。
Given that w, x, y and z are positive integers which satisfy the equation $w+x+y+z=12$. If there are W sets of different positive integral solutions of the equation, find the value of W.

7. 已知在數列 1001，1001001，1001001001，⋯，$1\underbrace{00}_{2}1\underbrace{00}_{2}1\ldots1\underbrace{00}_{2}1$，⋯ 中有 R 個質數，求 R 的值。
Given that the number of prime numbers in the sequence 1001, 1001001, 1001001001, ⋯ , $1\underbrace{00}_{2}1\underbrace{00}_{2}1\ldots1\underbrace{00}_{2}1$, ⋯ is R, find the value of R.

8. 設 $\lfloor x\rfloor$ 表示不大於 x 的最大整數，例如 $\lfloor 2.5\rfloor=2$。
若 $B=\left\lfloor\log_7\left(462+\log_2\lfloor\tan 60^\circ\rfloor+\sqrt{9872}\right)\right\rfloor$，求 B 的值。
Let $\lfloor x\rfloor$ be the largest integer not greater than x, for example, $\lfloor 2.5\rfloor=2$.
If $B=\left\lfloor\log_7\left(462+\log_2\lfloor\tan 60^\circ\rfloor+\sqrt{9872}\right)\right\rfloor$, find the value of B.

9. 已知 7^{2006} 的個位數是 C，求 C 的值。
Given that the units digit of 7^{2006} is C, find the value of C.

10. 如圖二，$ABCD$ 是一正方形，其邊長為 4 cm。綫段 PQ 和 MN 相交於點 O。若 PD 、 NC 、 BQ 和 AM 的長度是 1 cm, OQ 的長度是 x cm，求 x 的值。
In Figure 2, $ABCD$ is a square with side length equal to 4 cm. The line segments PQ and MN intersect at the point O. If the lengths of PD, NC, BQ and AM are 1 cm and the length of OQ is x cm, find the value of x.



圖二

Figure 2

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Hong Kong Mathematics Olympiad 2005-2006
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 設 a、b 和 c 是三個質數。若 $a < b < c$ 及 $c = a^2 + b^2$，求 a 的值。
Let a, b and c are three prime numbers. If $a < b < c$ and $c = a^2 + b^2$, find the value of a .

2. 若 $\log\left(\log\left(\log\left(1\overbrace{00...0}^{n\text{個}0}\right)\right)\right) = 1$，求 n 的值。
If $\log\left(\log\left(\log\left(1\overbrace{00...0}^{n\text{ zeros}}\right)\right)\right) = 1$, find the value of n .

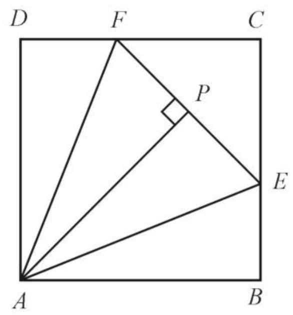
3. 已知 $0° < \theta < 90°$ 及 $1 + \sin\theta + \sin^2\theta + \cdots = \frac{3}{2}$。若 $y = \tan\theta$，求 y 的值。
Given that $0° < \theta < 90°$ and $1 + \sin\theta + \sin^2\theta + \cdots = \frac{3}{2}$. If $y = \tan\theta$, find the value of y.

4. 考慮二次方程 $x^2 - (a-2)x - a - 1 = 0$ ，其中 a 為實數。設 α 和 β 是方程的根。
求 a 的值使得 $\alpha^2 + \beta^2$ 的值最小。
Consider the quadratic equation $x^2 - (a-2)x - a - 1 = 0$, where a is a real number.
Let α and β be the roots of the equation.
Find the value of a such that the value of $\alpha^2 + \beta^2$ will be the least.

5. 已知連續 k 個正整數之和是 2006，求 k 最大可能的值。
Given that the sum of k consecutive positive integers is 2006,
find the maximum possible value of k .

6. 設 a、b、c 和 d 是實數且滿足 $a^2 + b^2 = c^2 + d^2 = 1$ 及 $ac + bd = 0$。
若 $R = ab + cd$，求 R 的值。
Let a, b, c and d be real numbers such that $a^2 + b^2 = c^2 + d^2 = 1$ and $ac + bd = 0$.
If $R = ab + cd$, find the value of R .

7. 如圖一，正方形 $ABCD$ 的周界是 16 cm，$\angle EAF = 45°$，$AP \perp EF$ 。
若 AP 的長度是 R m，求 R 的值。
In Figure 1, $ABCD$ is a square with perimeter equal to 16 cm, $\angle EAF = 45°$ and $AP \perp EF$.
If the length of AP is equal to R cm, find the value of R .



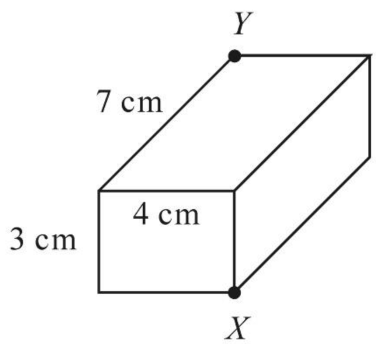
圖一
Figure 1

8. 已知 x 和 y 是實數且滿足方程組 $\begin{cases} \dfrac{100}{x+y} + \dfrac{64}{x-y} = 9 \\ \dfrac{80}{x+y} + \dfrac{80}{x-y} = 9 \end{cases}$，若 $V = x^2 + y^2$，求 V 的值。

Given that x and y are real numbers and satisfy the system of the equations

$\begin{cases} \dfrac{100}{x+y} + \dfrac{64}{x-y} = 9 \\ \dfrac{80}{x+y} + \dfrac{80}{x-y} = 9 \end{cases}$. If $V = x^2 + y^2$, find the value of V.

9. 如圖二，一長方體盒的邊長分別是 3 cm，4 cm 及 7 cm。
若在盒面上從點 X 到點 Y 的最短路徑的長度是 K cm，求 K 的值。
In Figure 2, given a rectangular box with dimensions 3 cm, 4 cm and 7 cm respectively.
If the length of the shortest path on the surface of the box from point X to point Y is K cm, find the value of K .



圖二
Figure 2

10. 已知 x 為正實數且滿足不等式 $|x-5| - |2x+3| \le 1$，求 x 的最小值。
Given that x is a positive real number which satisfy the inequality $|x-5| - |2x+3| \le 1$,
find the least value of x .

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