第二十四屆香港數學競賽(2006/2007)
初賽規則

1. 初賽分個人項目和團體項目兩部分。個人項目限時四十分鐘,團體項目限時二十分鐘。

2. 每隊由四至六位中四或以下同學組成。其中任何四位可參加個人項目;又其中任何四位可參加團體項目。不足四位同學的隊伍將不獲准出賽。

3. 每隊隊員必須穿著整齊校服,並由負責老師帶領,於比賽開始前十五分鐘向會場註冊處報到,否則大會有權取消該隊參賽資格。

4. 大會將以粵語發佈指示。若參賽者不諳粵語,則可獲發給一份中、英文指示。比賽題目則中、英文並列。

5. 個人項目中,每一隊員需要解答十條問題;而團體項目中,每一隊伍需要解答十條問題。

6. 團體項目中,各參賽隊員可進行討論,但必須將聲浪降至最低。

7. 比賽時,參賽者不可使用計算機、四位對數表等計算工具,違例隊伍將被取消資格或扣分。

8. 除另有聲明外,所有答案須為數字,並應化簡,但無需呈交證明及算草。

9. 參賽者如有攜帶傳呼機或手提電話,應把它關掉(包括響鬧功能)並放入手提包內或座椅之下。

10. 個人項目及團體項目中,每一正確答案均可得二分。每隊可得之最高積分為一百分。

11. 初賽的得分不包括快捷分。

12. 參賽者必須自備書寫工具,例如: 原子筆。

13. 按總成績選出最高積分的四十隊進入決賽。各分區最高積分的**五**隊將獲頒予獎狀。

14. 如有任何疑問,參賽者須於比賽完畢後,立即向會場主任提出。所提出之疑問,將由籌委會作最後裁決。

The Twenty-fourth Hong Kong Mathematics Olympiad (2006/2007)
Regulations (Heat Events)

1. The Heat Events consists of two parts: 40 minutes for individual event and 20 minutes for group event.

2. Each team should consist of 4 to 6 members who are students of Secondary 4 level or below. Any 4 of them may take part in the individual event and any 4 of them may take part in the group event. Teams of less than 4 members will not be allowed to particpate.

3. Members of each team, **accompanied by the teacher-in-charge, should wear proper school uniform** and report to the venue registrars 15 minutes before the commencement of the competition. Failing to do so, the team **will not be allowed** to particpate.

4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, written instructions in both Chinese and English will be provided. Question papers are printed in both Chinese and English.

5. Each member of a team has to solve 10 questions in the individual event and each team has to solve 10 questions in the group event.

6. In the group event, discussion among participating team members are allowed provided that the voice level is kept to a minimum.

7. Calculating devices, e.g. calculators, four-figure tables, will not be allowed to be used; otherwise the team will risk disqualification or deduction of marks.

8. All answers should be numerical and reduced to the simplest form unless stated otherwise. No proof or working is required.

9. If you have a pager or a mobile phone, you should turn it off (including the alarm function) and put it inside your bag or under your chair.

10. 2 marks will be given to each correct answer in both the individual event and the group event. The maximum score for a team should be 100.

11. No mark for speed will be awarded in the Heat Event.

12. Participants should bring along their own instruments, e.g. ball pens.

13. The 40 teams with the highest scores will be qualified for the Final Event.
    The first five teams in each region will be awarded certificates of Merits.

14. Any queries should reach the Centre Supervisor immediately after the competition. The decision of the Organising Committee on the queries is final.
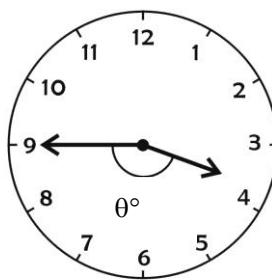
除非特別聲明，答案須用數字表達，並化至最簡。　　　　　　　　　　　時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark.　　　Time allowed: 40 minutes

1.　如圖一，時鐘顯示着三時四十五分。若時針與分針的交角是 θ°，求 θ 的值。

In Figure 1, a clock indicates the time 3: 45. If the angle between the hour-hand and the minute-hand is θ°, find the value of θ .
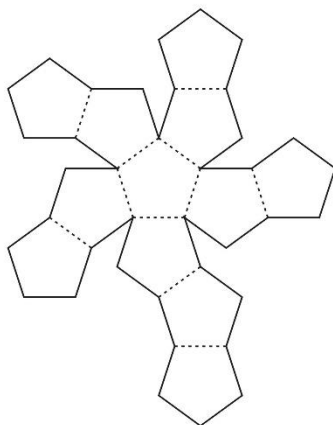
圖一
Figure 1

2.　如圖二的摺紙圖樣能摺出一個正多面體。若該正多面體有 y 條棱，求 y 的值。

In Figure 2, there is a paper net that can be wrapped into a regular polyhedron.
If this polyhedron has y edges, find the value of y .

圖二
Figure 2

3.　在 4 本英文書、6 本中文書及 9 本日文書中任取兩本。已知這兩本書是相同語言的。若有 X 個不同的選擇，求 X 的值。

Among 4 English books, 6 Chinese books and 9 Japanese books, two books are selected.
It is found that they are of the same language. If there are X such choices, find the value of X .

4.　設 r_1 和 r_2 是方程 $(x-2006)(x-2007)=2007$ 的兩個實根。
若 r 是方程 $(x-r_1)(x-r_2)=-2007$ 較小的實根，求 r 的值。

Let r_1 and r_2 be the two real roots of the equation $(x-2006)(x-2007)=2007$.
If r is the smaller real root of the equation $(x-r_1)(x-r_2)=-2007$, find the value of r .

5.　已知 α 及 β 是方程 $x^2-5^{2007}x+5^{1000}=0$ 的根。若 $s=\log_{25}\dfrac{\alpha^2}{\beta}+\log_{25}\dfrac{\beta^2}{\alpha}$，求 s 的值。

Given that α and β are the roots of the equation $x^2 - 5^{2007}x+5^{1000}=0$.

If $s=\log_{25}\dfrac{\alpha^2}{\beta}+\log_{25}\dfrac{\beta^2}{\alpha}$, find the value of s .

6.   對任意實數 a、b、c 及 d ，定義運算 $*$：$(a, b)*(c, d) = (ad + bc, bd)$。

若 $(x, y) = \left(1, \dfrac{3}{7 - \sqrt{5}}\right) * \left(8 + \sqrt{5}, 3\right)$ 及 $a = \dfrac{x}{y}$ ，求 a 的值。

For any real number a, b, c and d, we define the operation $*$: $(a, b)*(c, d) = (ad + bc, bd)$.

If $(x, y) = \left(1, \dfrac{3}{7 - \sqrt{5}}\right) * \left(8 + \sqrt{5}, 3\right)$ and $a = \dfrac{x}{y}$, find the value of a .

7.   已知 $\sin \alpha - \cos \alpha = \dfrac{1}{5}$ 及 $0° < \alpha < 180°$。若 $\tan \alpha = B$，求 B 的值。

Given that $\sin \alpha - \cos \alpha = \dfrac{1}{5}$ and $0° < \alpha < 180°$. If $\tan \alpha = B$, find the value of B .

8.   如圖三，ΔPAC、ΔQBA、ΔRCB 及 ΔABC 皆是等邊三角形。點 X、Y 及 Z 分別為 ΔPAC、ΔQBA 及 ΔRCB 的內心。若 PA 的長度是 10 cm 及 ΔXYZ 的周界是 w cm，求 w 的值。(註：三角形的內心為該三角形三條內角平分線的交點。)

In Figure 3, ΔPAC, ΔQBA, ΔRCB and ΔABC are equilateral triangles. The points X, Y and Z are the incentres of ΔPAC, ΔQBA and ΔRCB respectively. If the length of PA is 10 cm and the perimeter of ΔXYZ is w cm, find the value of w. (Remark: the incentre of a triangle is the point of intersection of the three interior angle bisectors of the triangle.)
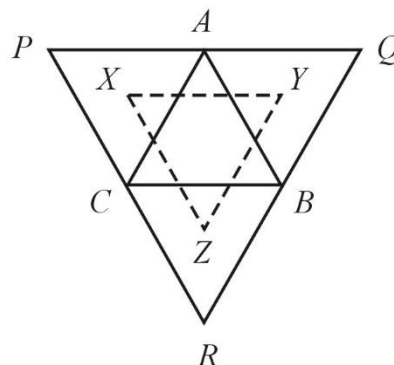
圖三
Figure 3

9.   設 $f(x) = \dfrac{1}{2}\left(4x^2 - 60x + 9 + \left|4x^2 - 60x + 9\right|\right)$ ，

若 $k = f(1) + f(2) + f(3) + \cdots + f(15) + f(16)$，求 k 的值。

Let $f(x) = \dfrac{1}{2}\left(4x^2 - 60x + 9 + \left|4x^2 - 60x + 9\right|\right)$.

If $k = f(1) + f(2) + f(3) + \cdots + f(15) + f(16)$, find the value of k .

10.   在平面上點 P 的坐標是 $(-3, 4)$。以 $(0, 0)$ 為中心，點 P 順時針方向旋轉 $45°$ 後，再沿 y-軸反射到達點 $Q = (x, y)$。若 $z = x + y$，求 z 的值。

The coordinates of point P on the plane is $(-3, 4)$. After rotating $45°$ clockwise about the centre $(0, 0)$ and reflecting along the y-axis, the point P reaches the point $Q = (x, y)$.

If $z = x + y$, find the value of z .

*** 試卷完 **End of Paper** ***

除非特別聲明，答案須用數字表達，並化至最簡。　　　　　　　　時限：20 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark.　　　Time allowed: 20 minutes

1. 若由 1 至 50 內與 50 互質的整數有 N 個，求 N 的值。(註：正整數 a 與 b 稱為互質若 a 與 b 的最大公因數是 1。)

   If there are N integers from 1 to 50 that are relatively prime to 50, find the value of N. (Remark: positive integers a and b are said to be relatively prime if their greatest common divisor is 1.)

2. 如圖一，在梯形 $ABCD$ 中，$AB /\!/ CD$，$\angle BCE = \angle ECD$，$CE \perp AD$ 及 $DE = 2AE$。若 ΔDEC 的面積是 $2007\,\text{cm}^2$ 及四邊形 $ABCE$ 的面積是 $T\,\text{cm}^2$，求 T 的值。

   In Figure 2, $ABCD$ is a trapezium, $AB /\!/ CD$, $\angle BCE = \angle ECD$, $CE \perp AD$ and $DE = 2AE$. If the area of ΔDEC is 2007 cm^2 and the area of quadrilateral $ABCE$ is T cm^2, find the value of T.
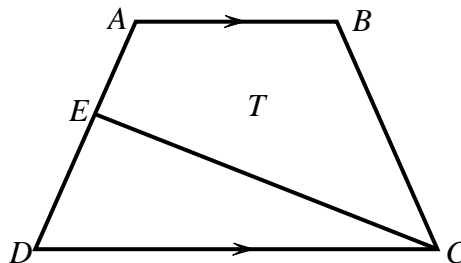
圖一
Figure 1

3. 已知 $a^2 - 3a + 1 = 0$。若 $A = \dfrac{2a^5 - 5a^4 + 2a^3 - 8a^2 + 7a}{3a^2 + 3}$，求 A 的值。

   Given that $a^2 - 3a + 1 = 0$. If $A = \dfrac{2a^5 - 5a^4 + 2a^3 - 8a^2 + 7a}{3a^2 + 3}$, find the value of A.

4. 已知點 A、B 及 C 的坐標分別為 $(3, 4)$、$(6, -4)$ 及 $(8, 10)$。M 及 N 分別為 AB 及 BC 的中點。X 為 AN 上一點使得 $AX : XN = 2 : 1$。若 $r = \dfrac{CX}{XM}$，求 r 的值。

   Given that the coordinates of the points A, B and C are $(3, 4)$, $(6, -4)$ and $(8, 10)$ respectively. M and N are the midpoints of AB and BC respectively. X is a point on AN such that $AX : XN = 2 : 1$. If $r = \dfrac{CX}{XM}$, find the value of r.

5. 如圖二，兩個邊長為 1 cm 的正方體組成一個 1 cm × 1 cm × 2 cm 的長方體。一隻螞蟻
    沿著長方體爬行，其爬行路綫須為正方體的棱。牠從頂點 A 出發，以每分鐘爬行 1 cm
    的速度，於 4 分鐘後到達頂點 B。若螞蟻可行路綫數目共有 S 個，求 S 的值。
    In Figure 2, a 1 cm × 1 cm × 2 cm rectangular box is made up by two cubes with side length
    1 cm. An ant is climbing along the box in a way that it must stay on the edges of the cubes
    through out the climbing. Starting from vertex A and climbing with a speed of 1 cm per minute,
    it reaches vertex B after 4 minutes. If the total number of possible paths taken by the ant is S,
    find the value of S.
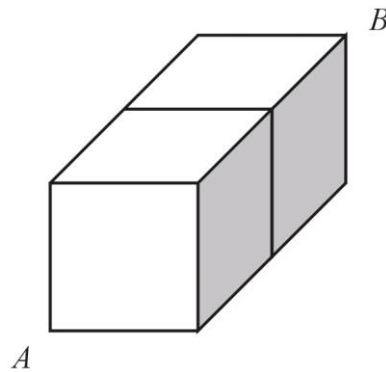
圖二
Figure 2

6. 若以 5 除 7^{2007} 所得的餘數是 R，求 R 的值。
    If the remainder of 7^{2007} when dividing by 5 is R, find the value of R.

7. 設 $k = \sin 30° + \cos 60° + \sin 90° + \cos 120° + \cdots + \sin 1890° + \cos 1920°$，求 k 的值。
    Let $k = \sin 30° + \cos 60° + \sin 90° + \cos 120° + \cdots + \sin 1890° + \cos 1920°$, find the value of k.

8. 如圖三，已知半圓的直徑為 10 cm。 A、B 和 C 是半圓上任意的三點使 B 在 $\overset{\frown}{AC}$ 上。
    設 x 為綫段 AB 及 BC 的長度之和，求 x 可取的最大值。
    In figure 3, given that the diameter of the semicircle is 10 cm. A, B and C are three arbitrary
    points on the semicircle where B is on $\overset{\frown}{AC}$. If x is the sum of the length of the line segments
    AB and BC, find the greatest possible value of x.

10 cm
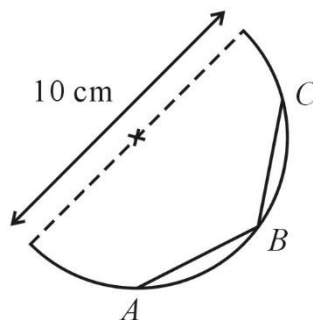
圖三
Figure 3

9. 在坐標平面上，點 $A = (-6, 2)$、$B = (-3, 3)$、$C = (0, n)$ 及 $D = (m, 0)$ 組成一個四邊形
    $ABCD$。求 n 的值使得該四邊形 $ABCD$ 的周界為最短。
    In the coordinate plane, the points $A = (-6, 2)$, $B = (-3, 3)$, $C = (0, n)$ and $D = (m, 0)$ form a
    quadrilateral $ABCD$.
    Find the value of n so that the perimeter of the quadrilateral $ABCD$ is the least.

10. 已知整數 x 及 y 滿足 $3x + 5y = 1$。若 $S = x - y$ 及 $S > 2007$，求 S 可取的最小值。
    Given that integers x and y satisfying the equation $3x + 5y = 1$.
    If $S = x - y$ and $S > 2007$, find the least possible value of S.

*** 試卷完 End of Paper ***

初賽成績（一）
初賽日期：2007 年 1 月 27 日星期六
試場一(HK)：
試場二(KLN)：
試場三(NTE)：
試場四(NTW)：

List of Participants Obtainded Full Score in the Individual Event

Name of School	Name of Student
La Salle College	WONG Sing-hung
	TANG Man-lok
STFA Leung Kau Kui College	NG Ngai-fung

List of Schoola Obtained Full Score in the Group Event
1.   La Salle College
2.   Cheung Chuk Shan College

Regional winners of the Heat Event
Hong Kong Region

Position	School Name
Winner	Cheung Chuk Shan College
1st Runner-up	Ying Wa Girls' School
2nd Runner-up	King's College
	Pui Kiu Middle School
5th Place	Queen's College
	St. Mark's School

Kowloon Region

Position	School Name
Winner	La Salle College
1st Runner-up	Kwun Tong Government Secondary School
2nd Runner-up	Diocesan Boys' School
4th Place	The Bishop Hall Jubilee School
5th Place	Fukien Secondary School

New Territories East Region

Position	School Name
Winner	Immaculate Heart of Mary College
1st Runner-up	Carmel Pak U Secondary School
	SKH Tsang Shiu Tim Secondary School
	TWGH Kap Yan Directors' College
2nd Runner-up	Baptist Lui Ming Choi Secondary School
	SHK Bishop Mok Sau Tseng Secondary School

New Territories West Region

Position	School Name
Winner	Shung Tak Catholic English College
1st Runner-up	STFA Leung Kau Kui College
2nd Runner-up	Tsuen Wan Government Secondary School
4th Place	PLK Centary Li Shiu Chung Memorial College
5th Place	Ho Fung College (Sponsored by Sik Sik Yuen)
	Yuen Long Merchants Association Secondary School

初賽成績（二）決賽名單：

School ID	Heat ID	Name of School
FE-01	NT1-02	Baptist Lui Ming Choi Secondary School
FE-02	HK-03	Belilios Public School
FE-03	KLN1-01	Bishop Hall Jubilee School
FE-04	NT1-06	Carmel Pak U Secondary School
FE-05	HK-05	Cheung Chuk Shan College
FE-06	KLN2-07	Choi Hung Estate Catholic Secondary School
FE-07	KLN1 -09	Diocesan Boys' School
FE-08	KLN2-10	Fukien Secondary School
FE-09	KLN2-11	Good Hope School
FE-10	NT2-15	Ho Fung College (Sponsored by Sik Sik Yuen)
FE-11	NT1-20	HKTA Tang Hin Memorial Secondary School
FE-12	HK-13	Hong Kong Chinese Women's Club College
FE-13	NT1-21	Immaculate Heart of Mary College
FE-14	KLN2-18	King Ling College
FE-15	HK-17	King's College
FE-16	KLN2-20	Kwun Tong Government Secondary School
FE-17	KLN2-22	Kwun Tong Maryknoll College
FE-18	KLN1-23	La Salle College
FE-19	NT2-27	PLK Centenary Li Shiu Chung Memorial College
FE-20	KLN2-28	PLK No. 1 WH Cheung College
FE-21	KLN1-34	Pui Ching Middle School
FE-22	HK-22	Pui Kiu Middle School
FE-23	HK-24	Queen's College
FE-24	HK-28	San Wui Commercial Society Chan Pak Sha School
FE-25	NT1-37	Shatin Tsung Tsin Secondary School
FE-26	NT2-33	Shung Tak Catholic English College
FE-27	KLN2-33	Sing Ying Secondary School
FE-28	NT1-40	SKH Bishop Mok Sau Tseng Secondary School
FE-29	NT2-34	SKH Lam Woo Memorial Secondary School
FE-30	HK-31	SKH Lui Ming Choi Secondary School
FE-31	NT1-42	SKH Tsang Shiu Tim Secondary School
FE-32	HK-37	St Mark's School
FE-33	HK-38	St Paul Co-educational College
FE-34	NT2-40	STFA Leung Kau Kui College
FE-35	NT2-44	Tsuen Wan Government Secondary School
FE-36	NT1-48	TWGH Kap Yan Ditrectors' College
FE-37	KLN1 -47	Wah Yan College (Kowloon)
FE-38	KLN1-49	Ying Wa College
FE-39	HK-46	Ying Wa Girls' School
FE-40	NT2-54	Yuen Long Merchants Association Secondary School