Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event Sample (Individual)

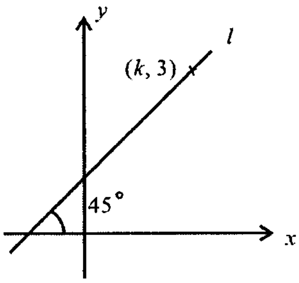
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $\sqrt{k}=\sqrt{7+\sqrt{13}}-\sqrt{7-\sqrt{13}}$，求 k 的值。
Let $\sqrt{k}=\sqrt{7+\sqrt{13}}-\sqrt{7-\sqrt{13}}$, find the value of k.

$k =$

2. 如圖一，直綫 ℓ 經過點 $(k, 3)$ 並與 x 軸成 45° 夾角。若 ℓ 的方程是 $x+by+c=0$ 及 $b=|1+b+c|$，求 d 的值。
In Figure 1, the straight line ℓ passes though the point $(k, 3)$ and makes an angle 45° with the x-axis.
If the equation of ℓ is $x+by+c=0$ and $d=|1+b+c|$, find the value of d.

$d =$



圖一 Figure 1

3. 若 $x-d$ 為 $x^3-6x^2+11x+a$ 的因式,，求 a 的值。
If $x-d$ is a factor of $x^3-6x^2+11x+a$, find the value of a.

$a =$

4. 若 $\cos x+\sin x=-\frac{a}{5}$ 及 $t=\tan x+\cot x$，求 t 的值。
If $\cos x+\sin x=-\frac{a}{5}$ and $t=\tan x+\cot x$, find the value of t.

$t =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $A = 15\times\tan 44°\times\tan 45°\times\tan 46°$，求 A 的值。
   Let $A = 15\times\tan 44°\times\tan 45°\times\tan 46°$, find the value of A.

   $A =$

2. 設 n 為正整數及 $\overbrace{20082008\cdots2008}^{n\text{個}2008}15$ 能被 A 整除。
   若 n 的最小可能值是 B，求 B 的值。
   Let n be a positive integer and $\overbrace{20082008\cdots2008}^{n\ 2008\text{'s}}15$ is divisible by A.
   If the least possible value of n is B, find the value of B.

   $B =$

3. 已知有 C 個整數滿足方程 $|x-2|+|x+1|=B$，求 C 的值。
   Given that there are C integers that satisfy the equation $|x-2|+|x+1|=B$, find the value of C.

   $C =$

4. 在座標平面上，點 $(-C, 0)$ 與直綫 $y=x$ 的距離是 $\sqrt{D}$，求 D 的值。
   In the coordinate plane, the distance from the point $(-C, 0)$ to the straight line $y=x$ is $\sqrt{D}$, find the value of D.

   $D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $P=\left[\sqrt[3]{6}\times\left(\sqrt[3]{\frac{1}{162}}\right)\right]^{-1}$，求 P 的值。

Given that $P=\left[\sqrt[3]{6}\times\left(\sqrt[3]{\frac{1}{162}}\right)\right]^{-1}$, find the value of P.

$P=$

2. 設 a、b 和 c 是實數且 $b:(a+c)=1:2$ 及 $a:(b+c)=1:P$。
若 $Q=\frac{a+b+c}{a}$，求 Q 的值。
Let a, b and c be real numbers with ratios $b:(a+c)=1:2$ and $a:(b+c)=1:P$.
If $Q=\frac{a+b+c}{a}$, find the value of Q.

$Q=$

3. 設 $R=\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)^{Q}+\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)^{Q}$。求 R 的值。

Let $R=\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)^{Q}+\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)^{Q}$. Find the value of R.

$R=$

4. 設 $S=(x-R)^2+(x+5)^2$，其中 x 為實數。求 S 的最小值。
Let $S=(x-R)^2+(x+5)^2$, where x is a real number. Find the minimum value of S.

$S=$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

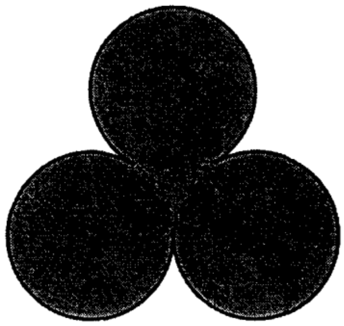
1. 已知 $\frac{1-\sqrt{3}}{2}$ 滿足方程 $x^2+px+q=0$，其中 p 和 q 是有理數。
若 $A=|p|+2|q|$，求 A 的值。
Given that $\frac{1-\sqrt{3}}{2}$ satisfies the equation $x^2+px+q=0$, where p and q are rational numbers. If $A=|p|+2|q|$, find the value of A.

$A=$

2. U_1 及 U_2 兩袋有大小相同的紅球和白球。U_1 裝有 A 個紅球，2 個白球。U_2 裝有 2 個紅球，B 個白球。從每袋中各取出 2 個球。
若取到四個紅球的概率是 $\frac{1}{60}$，求 B 的值。
Two bags U_1 and U_2 contain identical red and white balls. U_1 contains A red balls and 2 white balls. U_2 contains 2 red balls and B white balls. Take two balls out of each bag. If the probability of all four balls are red is $\frac{1}{60}$, find the value of B.

$B=$

3. 圖一由三個大小相同互切的圓所組成，三個圓的半徑均是 B cm。
若陰影部分的周界是 C cm，求 C 的值。(取 $\pi=3$)
Figure 1 is formed by three identical circles touching one another, the radius of each circle is B cm. If the perimeter of the shaded region is C cm, find the value of C. (Take $\pi=3$)



圖一 Figure 1

$C=$

4. 設與 $\sqrt{C}$ 最接近的整數是 D，求 D 的值。
Let D be the integer closest to $\sqrt{C}$, find the value of D.

$D=$

FOR OFFICIAL USE

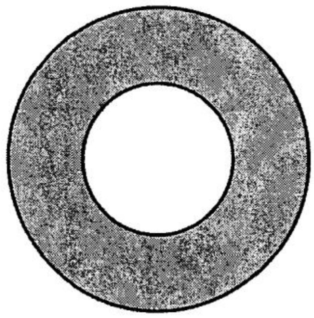
Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+ Bonus score				Time		
			Total score					Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 x 及 y 為實數，且滿足$|x|+x+y=10$ 及 $|y|+x-y=10$。
若 $P=x+y$，求 P 的值。
Given that x and y are real numbers such that $|x|+x+y=10$ and $|y|+x-y=10$.
If $P=x+y$, find the value of P.

$P=$

2. 如圖一，陰影部分由兩同心圓所組成，其面積為 $96\pi\text{ cm}^2$。若該兩圓的半徑相差 $2P$ cm 及大圓的面積為 $Q\text{ cm}^2$，求 Q 的值。(取 $\pi=3$)
In Figure 1, the shaded area is formed by two concentric circles and has area $96\pi\text{ cm}^2$. If the two radii differ by $2P$ cm and the large circle has area $Q\text{ cm}^2$, find the value of Q. (Take $\pi=3$)



圖一 Figure 1

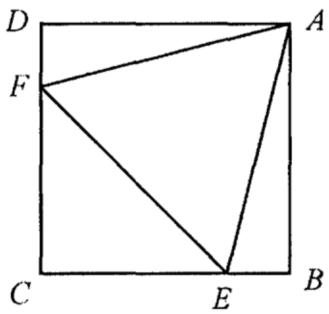
$Q=$

3. 設 R 為最大的整數使得 $R^Q<5^{200}$ 成立，求 R 的值。
Let R be the largest integer such that $R^Q<5^{200}$, find the value of R.

$R=$

4. 圖二顯示一個邊長為 $(R-1)$ cm 的正方形 $ABCD$ 及一個等邊三角形 AEF (E 及 F 分別是直綫 BC 及 CD 上的點)。若 ΔAEF 的面積是$(S-3)\text{ cm}^2$，求 S 的值。
In Figure 2, there are a square $ABCD$ with side length $(R-1)$ cm and an equilateral triangle AEF. (E and F are points on BC and CD respectively).
If the area of ΔAEF is $(S-3)\text{ cm}^2$, find the value of S.

$S=$



圖二 Figure 2

FOR OFFICIAL USE

Score for accuracy $\times$ Mult. factor for speed $=$

$+$ Bonus score

Total score

Team No.

Time Min. Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event Spare (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 28 的所有正因子是 $d_1, d_2, \ldots, d_n$ 及 $a=\frac{1}{d_1}+\frac{1}{d_2}+\cdots+\frac{1}{d_n}$，求 a 的值。

   If all the positive factors of 28 are $d_1, d_2, \ldots, d_n$ and $a=\frac{1}{d_1}+\frac{1}{d_2}+\cdots+\frac{1}{d_n}$, find the value of a.

   $a =$

2. 已知 x 為負實數且 $\dfrac{1}{x+\dfrac{1}{x+2}}=a$。若 $b=x+\frac{7}{2}$，求 b 的值。

   Given that x is a negative real number that satisfy $\dfrac{1}{x+\dfrac{1}{x+2}}=a$.

   If $b=x+\frac{7}{2}$, find the value of b.

   $b =$

3. 設 α 和 β 是方程 $x^2+cx+b=0$ 的兩個根，其中 $c<0$ 及 $\alpha-\beta=1$。求 c 的值。

   Let α and β be the two roots of the equation $x^2 + cx + b = 0$, where $c < 0$ and $\alpha - \beta = 1$. Find the value of c.

   $c =$

4. 設 d 為$(196c)^{2008}$ 除以 97 所得的餘數。求 d 的值。

   Let d be the remainder of $(196c)^{2008}$ divided by 97. Find the value of d.

   $d =$

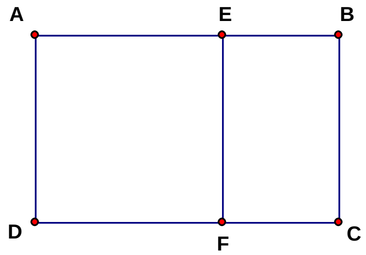
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
			Total score					Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，$AEFD$ 是邊長為一單位的正方形。長方形 $ABCD$ 的長闊的比例與長方形 $BCFE$ 的長闊比例相同。若 AB 的長度是 W 單位，求 W 的值。
In Figure 1, $AEFD$ is a unit square. The ratio of the length of the rectangle $ABCD$ to its width is equal to the ratio of the length of the rectangle $BCFE$ to its width.
If the length of AB is W units, find the value of W.

$W =$



圖一
Figure 1

2. 在座標平面上滿足 $x^2 + y^2 < 10$，其中 x 及 y 為整數的點(x, y)共有 T 個，求 T 的值。
On the coordinate plane, there are T points (x, y), where x, y are integers, satisfying $x^2 + y^2 < 10$, find the value of T.

$T =$

3. 設 P 及 $P + 2$ 均為質數並滿足 $P(P + 2) \le 2007$。
若 S 是符合上述要求的質數 P 的總和，求 S 的值。
Let P and $P + 2$ be both prime numbers satisfying $P(P + 2) \le 2007$.
If S represents the sum of such possible values of P, find the value of S.

$S =$

4. 已知 $\log_{10}(2007^{2006} \times 2006^{2007}) = a\times10^k$，其中 $1 \le a < 10$ 及 k 是整數，求 k 的值。
It is known that $\log_{10}(2007^{2006} \times 2006^{2007}) = a\times10^k$, where $1 \le a < 10$ and k is an integer. Find the value of k.

$k =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 1 (Group)

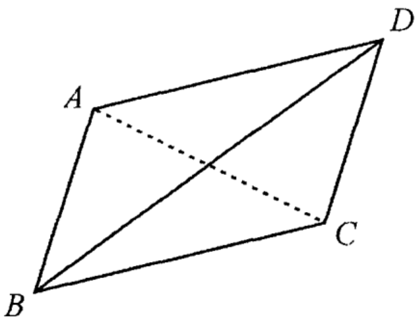
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知座標平面上三點：$O(0,0)$、$A(12,2)$ 及 $B(0,8)$。 ΔOAB 經直綫 $y=6$ 作反射後得ΔPQR。若ΔOAB 及ΔPQR 重疊部分的面積是 m 平方單位，求 m 的值。
Given that there are three points on the coordinate plane: $O(0, 0)$, $A(12, 2)$ and $B(0, 8)$. A reflection of ΔOAB along the straight line $y = 6$ creates ΔPQR. If the overlapped area of ΔOAB and ΔPQR is m square units, find the value of m.

$m =$

2. 如圖一，$ABCD$ 是平行四邊形，$BA = 3$ cm、$BC = 4$ cm 及 $BD = \sqrt{37}$ cm。若 $AC = h$ cm，求 h 的值。
In Figure 1, $ABCD$ is a parallelogram with $BA = 3$ cm, $BC = 4$ cm and $BD = \sqrt{37}$ cm. If $AC = h$ cm, find the value of h.

$h =$



圖一
Figure 1

3. 已知 x、y 及 z 為正整數及分數 $\frac{151}{44}$ 可寫成 $3+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$ 的形式。求 $x+y+z$ 的值。
Given that x, y and z are positive integers and the fraction $\frac{151}{44}$ can be written in the form of $3+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$. Find the value of $x+y+z$.

$z =$

4. 當 491 除以一個兩位數，餘數是 59。求這兩位數。
When 491 is divided by a two-digit integer, the remainder is 59.
Find this two-digit integer.

FOR OFFICIAL USE

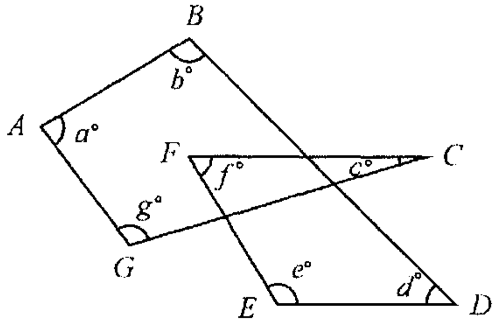
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，BD、FC、GC 及 FE 為直綫。若 $z = a + b + c + d + e + f + g$，求 z 的值。
In Figure 1, BD, FC, GC and FE are straight lines.
If $z = a + b + c + d + e + f + g$, find the value of z.

$z =$



圖一

Figure 1

2. 若 $1^6 + 2^6 + 3^6 + 4^6 + 5^6 + 6^6$ 被 7 除後的餘數是 R，求 R 的值。
If R is the remainder of $1^6 + 2^6 + 3^6 + 4^6 + 5^6 + 6^6$ divided by 7, find the value of R.

$R =$

3. 若 14! 能被 6^k 整除，其中 k 為整數，求 k 的最大可能值。
If 14! is divisible by 6^k, where k is an integer, find the largest possible value of k.

$k =$

4. 設實數 x、y 及 z 滿足 $x + \frac{1}{y} = 4$，$y + \frac{1}{z} = 1$ 及 $z + \frac{1}{x} = \frac{7}{3}$。求 xyz 的值。
Let x, y and z be real numbers that satisfy $x + \frac{1}{y} = 4$, $y + \frac{1}{z} = 1$ and $z + \frac{1}{x} = \frac{7}{3}$.
Find the value of xyz.

$xyz =$

FOR OFFICIAL USE

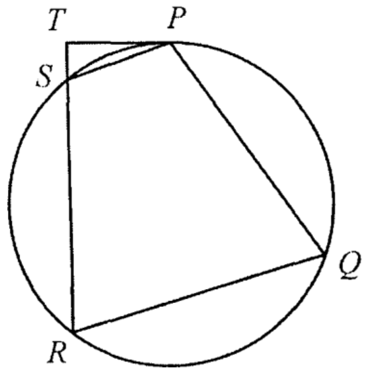
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖一，$PQRS$ 是一個圓內接四邊形，其中 S 在直綫 RT 上且 TP 為該圓的切綫。若 $RS = 8$ cm，$RT = 11$ cm 及 $TP = k$ cm，求 k 的值。
In Figure 1, $PQRS$ is a cyclic quadrilateral, where S is on the straight line RT and TP is tangent to the circle. If $RS = 8$ cm, $RT = 11$ cm and $TP = k$ cm, find the value of k .

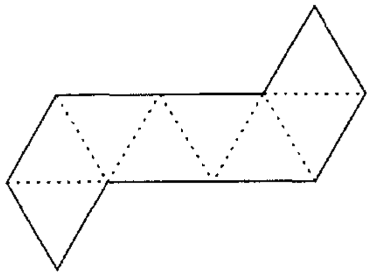
$k =$



圖一
Figure 1

2. 圖二中的摺紙圖樣能摺出一多面體。若該多面體有 v 個頂點，求 v 的值。
The layout in Figure 2 can be used to fold a polyhedron.
If this polyhedron has v vertices, find the value of v .

$v =$



圖二
Figure 2

3. 對任意實數 x，定義$[x]$是小於或等於 x 的最大整數。例如，$[2] = 2$，$[3.4] = 3$。求 $[1.008^8 \times 100]$ 的值。
For arbitrary real number x, define $[x]$ to be the largest integer less than or equal to x. For instance, $[2] = 2$ and $[3.4] = 3$. Find the value of $[1.008^8 \times 100]$.

4. 當從標明了 1 至 30 的 30 個號碼球中選出 4 個，而選出的球均不放回重選時，能得 r 個組合，求 r 的值。
When choosing, without replacement, 4 out of 30 labelled balls that are marked from 1 to 30, there are r combinations. Find the value of r .

$r =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 利用相同的正 m 邊形能密鋪平面，求所有可能 m 值的總和。
Regular tessellation is formed by identical regular m-polygons for some fixed m.
Find the sum of all possible values of m .

sum of $m =$

2. 在 3624、36024、360924、3609924、36099924、360999924 及 3609999924 這七個數中，能被 38 整除的有 n 個，求 n 的值。
Amongst the seven numbers 3624, 36024, 360924, 3609924, 36099924, 360999924 and 3609999924, there are n of them that are divisible by 38.
Find the value of n .

$n =$

3. 若 $208208 = 8^5a + 8^4b + 8^3c + 8^2d + 8e + f$，其中 a、b、c、d、e 及 f 為整數且 $0 \le a, b, c, d, e, f \le 7$，求 $a\times b\times c + d\times e\times f$ 的值。
If $208208 = 8^5a + 8^4b + 8^3c + 8^2d + 8e + f$, where a, b, c, d, e, and f are integers and $0 \le a, b, c, d, e, f \le 7$, find the value of $a\times b\times c + d\times e\times f$.

4. 在座標平面上，點 $A(6, 8)$ 繞原點 $O(0, 0)$逆時針轉 20070° 至點 $B(p, q)$。求 $p + q$ 的值。
In the coordinate plane, rotate point $A(6, 8)$ about the origin $O(0, 0)$ counter-clockwise for 20070° to point $B(p, q)$. Find the value of $p + q$.

$p + q =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event Spare (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 計算 $\left(\sqrt{2008}+\sqrt{2007}\right)^{2007}\times\left(\sqrt{2007}-\sqrt{2008}\right)^{2007}$ 的值。
   Calculate the value of $\left(\sqrt{2008}+\sqrt{2007}\right)^{2007}\times\left(\sqrt{2007}-\sqrt{2008}\right)^{2007}$.

2. 若 $x-\frac{1}{x}=\sqrt{2007}$，求 $x^4+\frac{1}{x^4}$ 的值。
   If $x-\frac{1}{x}=\sqrt{2007}$, find the value of $x^4+\frac{1}{x^4}$.

3. 已知 $\cos\alpha=-\frac{99}{101}$ 及 $180°<\alpha<270°$。求 $\cot\alpha$ 的值。
   Given that $\cos\alpha=-\frac{99}{101}$ and $180°<\alpha<270°$. Find the value of $\cot\alpha$.

4. 求 $\frac{2008^3+4015^3}{2007^3+4015^3}$ 的值。
   Calculate the value of $\frac{2008^3+4015^3}{2007^3+4015^3}$.

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.