Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event Sample (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設$[x]$是不超過x的最大整數。若$a=\left[\left(\sqrt{3}-\sqrt{2}\right)^{2009}\right]+16$，求$a$的值。

   Let $[x]$ be the largest integer not greater than x.

   If $a=\left[\left(\sqrt{3}-\sqrt{2}\right)^{2009}\right]+16$, find the value of a.

   $a=$

2. 在座標平面上，若x-軸、y-軸與直綫 $3x+ay=12$ 所圍成三角形的面積是 b 平方單位，求 b 的值。

   In the coordinate plane, if the area of the triangle formed by the x-axis, y-axis and the line $3x+ay=12$ is b square units, find the value of b.

   $b=$

3. 已知$x-\frac{1}{x}=2b$及$x^3-\frac{1}{x^3}=c$，求c的值。

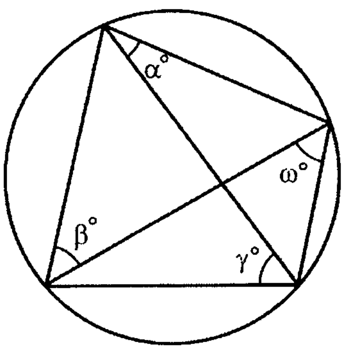
   Given that $x-\frac{1}{x}=2b$ and $x^3-\frac{1}{x^3}=c$, find the value of c.

   $c=$

4. 如圖一，$\alpha=c$、$\beta=43$、$\gamma=59$ 及 $\omega=d$，求d的值。

   In Figure 1, $\alpha=c$, $\beta=43$, $\gamma=59$ and $\omega=d$, find the value of d.

   $d=$



圖一
Figure 1

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

+ Bonus score

Time

Total score

Min. Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 把三個體積分別為 1、8、27 的正立方體，以面同貼面的方法黏合起來。
若 a 為所得的多面體的最小總表面積，求 a 的值。
Three cubes with volumes 1, 8, 27 are glued together at their faces.
If a is the smallest possible surface area of the resulting polyhedron, find the value of a.

$a =$

2. 已知 $f(x) = -x^2 + 10x + 9$，且 $2 \le x \le \frac{a}{9}$。若 b 是 f 的最大及最小值之差，求 b 的值。
Given that $f(x) = -x^2 + 10x + 9$, and $2 \le x \le \frac{a}{9}$.
If b is the difference of the maximum and minimum values of f, find the value of b .

$b =$

3. 已知 p 及 q 是實數，且 $pq = b$ 及 $p^2q + q^2p + p + q = 70$。若 $c = p^2 + q^2$，求 c 的值。
Given that p and q are real numbers with $pq = b$ and $p^2q + q^2p + p + q = 70$.
If $c = p^2 + q^2$, find the value of c .

$c =$

4. 在一個有 c 行的演奏廳中，每一行都比前一行多兩個座位。
若中間的行有 64 個座位，這演奏廳共有多少個座位 (d)？
There are c rows in a concert hall and each succeeding row has two more seats than the previous row.
If the middle row has 64 seats, how many seats (d) does the concert have ?

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 a，p，q 是質數，且滿足 $a < p$ 及 $a + p = q$，求 a 的值。
If a, p, q are primes with $a < p$ and $a + p = q$, find the value of a.

$a =$

2. 若 b 及 h 為正整數，且滿足 $b < h$ 及 $b^2 + h^2 = b(a + h) + ah$，求 b 的值。
If b and h are positive integers with $b < h$ and $b^2 + h^2 = b(a + h) + ah$, find the value of b.

$b =$

3. 在一個$(2b + 1)\times(2b + 1)$的棋盤上任意選取兩個不在同一橫行上方格。若 c 為選取的兩個不同方格的組合數目，求 c 的值。
In a $(2b + 1)\times(2b + 1)$ checkerboard, two squares not lying in the same row are randomly chosen. If c is the number of combinations of different pairs of squares chosen, find the value of c.

$c =$

4. 已知 $f(x) = c\left|\frac{1}{x} - \left\lfloor \frac{1}{x} + \frac{1}{2} \right\rfloor\right|$，其中$\lfloor x \rfloor$是小於或等於實數 x 的最大整數。若 d 為 $f(x)$的最大值，求 d 的值。
Given that $f(x) = c\left|\frac{1}{x} - \left\lfloor \frac{1}{x} + \frac{1}{2} \right\rfloor\right|$，where $\lfloor x \rfloor$ is the greatest integer less than or equal to the real number x. If d is the maximum value of $f(x)$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 a 為 15147 的相異質因數的數目。求 a 的值。
If a is the number of distinct prime factors of 15147, find the value of a.

$a =$

2. 若 $x+\frac{1}{x}=a$ 及 $x^3+\frac{1}{x^3}=b$，求 b 的值。
If $x+\frac{1}{x}=a$ and $x^3+\frac{1}{x^3}=b$, find the value of b.

$b =$

3. 設 $\mathrm{f}(x)=\begin{cases} x+5 & \text{當 } x \text{ 是一奇數} \\ \frac{x}{2} & \text{當 } x \text{ 是一偶數} \end{cases}$。
若 c 是一奇數及 $\mathrm{f}(\mathrm{f}(\mathrm{f}(c)))=b$，求 c 的最小值。
Let $\mathrm{f}(x)=\begin{cases} x+5 & \text{if } x \text{ is an odd integer} \\ \frac{x}{2} & \text{if } x \text{ is an even integer} \end{cases}$.
If c is an odd integer and $\mathrm{f}(\mathrm{f}(\mathrm{f}(c)))=b$, find the least value of c.

$c =$

4. 設 $\mathrm{f}\left(\frac{x}{3}\right)=x^2+x+1$。若 d 為所有滿足 $\mathrm{f}(3x)=c$ 的 x 之和，求 d 的值。
Let $\mathrm{f}\left(\frac{x}{3}\right)=x^2+x+1$. If d is the sum of all x for which $\mathrm{f}(3x)=c$, find the value of d.

$d =$

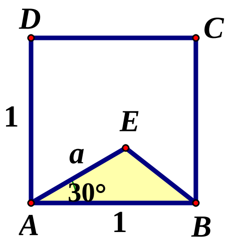
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 在圖一中，$ABCD$ 為一正方形，E 為一點及 $\angle EAB = 30^\circ$。若 $ABCD$ 的面積是 ΔABE 的面積的六倍，則 $AE : AB = a : 1$。求 a 的值。
In Figure 1, $ABCD$ is a square, E is a point and $\angle EAB = 30^\circ$. If the area of $ABCD$ is six times that of ΔABE, then the ratio of $AE : AB = a : 1$. Find the value of a.



圖一
Figure 1

$a =$

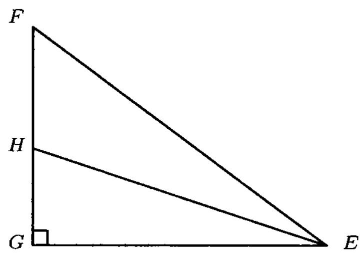
2. 已知 $b = \dfrac{\log 8^a + \log 27^a + \log 125^a}{\log 9 + \log 25 + \log 2 - \log 15}$，求 b 的值。
Given that $b = \dfrac{\log 8^a + \log 27^a + \log 125^a}{\log 9 + \log 25 + \log 2 - \log 15}$, find the value of b.

$b =$

3. 設 c 為 $1^3 + 2^3 + \ldots + 2009^3 + 2010^3$ 除以 b^2 的餘數，求 c 的值。
Let c be the remainder of $1^3 + 2^3 + \ldots + 2009^3 + 2010^3$ divided by b^2, find the value of c.

$c =$

4. 在圖二中，EFG 為一直角三角形。已知 H 為 FG 上的一點，使得 $GH : HF = 4 : 5$ 及 $\angle GEH = \angle FEH$。若 $EG = c$ 及 $FG = d$，求 d 的值。
In Figure 2, EFG is a right-angled triangle.
Given that H is a point on FG, such that $GH : HF = 4 : 5$ and $\angle GEH = \angle FEH$.
If $EG = c$ and $FG = d$, find the value of d.



圖二
Figure 2

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
			+	Bonus score			Time
				Total score			Min. / Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event Spare (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $a=\sqrt{(19.19)^2+(39.19)^2-(38.38)(39.19)}$。求 a 的值。

   Given that $a=\sqrt{(19.19)^2+(39.19)^2-(38.38)(39.19)}$. Find the value of m.

   $a =$

2. 給定四點 $R(0, 0)$、$S(a, 0)$、$T(a, 6)$及 $U(0, 6)$。
   若直綫 $y = b(x - 7) + 4$ 把四邊形 $RSTU$ 分成兩份，其面積相等，求 b 的值。
   Given four points $R(0, 0)$, $S(a, 0)$, $T(a, 6)$ and $U(0, 6)$. If the line $y = b(x - 7) + 4$ cuts the quadrilateral $RSTU$ into two halves of equal area, find the value of b.

   $b =$

3. 已知 c 為 $\mathrm{f}(x)=\dfrac{x^2-2x+\frac{1}{b}}{2x^2+2x+1}$ 的最小值。求 c 的值。

   Given that c is the minimum value of $\mathrm{f}(x)=\dfrac{x^2-2x+\frac{1}{b}}{2x^2+2x+1}$. Find the value of c.

   $c =$

4. 已知 $\mathrm{f}(x) = px^6 + qx^4 + 3x - \sqrt{2}$，且 p、q 為非零實數。
   若 $d = \mathrm{f}(c) - \mathrm{f}(-c)$，求 d 的值。
   Given that $\mathrm{f}(x) = px^6 + qx^4 + 3x - \sqrt{2}$, and p, q are non-zero real numbers.
   If $d = \mathrm{f}(c) - \mathrm{f}(-c)$, find the value of d.

   $d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event Sample (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $\tan\theta = \dfrac{5}{12}$，其中 $180° \le \theta \le 270°$。若 $A = \cos\theta + \sin\theta$，求 A 的值。
Given $\tan\theta = \dfrac{5}{12}$, where $180° \le \theta \le 270°$.
If $A = \cos\theta + \sin\theta$, find the value of A.

$A =$

2. 設 $[x]$是不超過 x 的最大整數。若 $B = \left[10+\sqrt{10+\sqrt{10+\sqrt{10+\cdots}}}\right]$，求 B 的值。
Let $[x]$ be the largest integer not greater than x.
If $B = \left[10+\sqrt{10+\sqrt{10+\sqrt{10+\cdots}}}\right]$, find the value of B.

$B =$

3. 設 $a \oplus b = ab + 10$。若 $C = (1\oplus 2) \oplus 3$，求 C 的值。
Let $a \oplus b = ab + 10$. If $C = (1\oplus 2) \oplus 3$, find the value of C.

$C =$

4. 在座標平面上，用以下直綫所圍成圖形的面積為 D 平方單位，求 D 的值。

$$L_1: y - 2 = 0$$
$$L_2: y + 2 = 0$$
$$L_3: 4x + 7y - 10 = 0$$
$$L_4: 4x + 7y + 20 = 0$$

In the coordinate plane, the area of the region bounded by the following lines is D square units, find the value of D.

$$L_1: y - 2 = 0$$
$$L_2: y + 2 = 0$$
$$L_3: 4x + 7y - 10 = 0$$
$$L_4: 4x + 7y + 20 = 0$$

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 求 $\sin^2 1° + \sin^2 2° + \ldots + \sin^2 89°$ 的值。
Find the value of $\sin^2 1° + \sin^2 2° + \ldots + \sin^2 89°$.

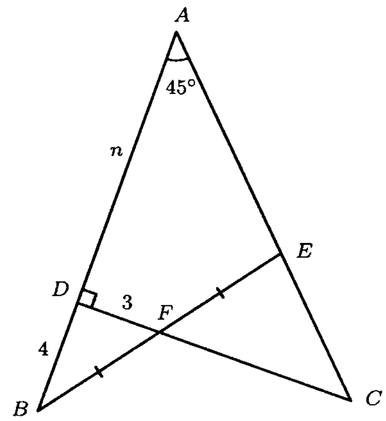
2. 已知 $\frac{x+z}{2z-x} = \frac{z+2y}{2x-z} = \frac{x}{y}$，其中 x、y、z 為正數。求 $\frac{x}{y}$ 的值。
Given that $\frac{x+z}{2z-x} = \frac{z+2y}{2x-z} = \frac{x}{y}$, where x, y and z are positive numbers.
Find the value of $\frac{x}{y}$.

$\frac{x}{y} =$

3. 求方程 $(2^x - 4)^3 + (4^x - 2)^3 = (4^x + 2^x - 6)^3$ 的所有實根 x 的總和。
Find the sum of all real roots x of the equation $(2^x - 4)^3 + (4^x - 2)^3 = (4^x + 2^x - 6)^3$.

4. 在圖一，若 $AB \perp CD$，F 是 BE 的中點，$\angle A = 45°$，$DF = 3$, $BD = 4$ 及 $AD = n$，求 n 的值。
In Figure 1, if $AB \perp CD$, F is the midpoint of BE, $\angle A = 45°$, $DF = 3$, $BD = 4$ and $AD = n$, find the value of n.

$n =$



圖一
Figure 1

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
			Total score					Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $p = 2 - 2^2 - 2^3 - 2^4 - \ldots - 2^9 - 2^{10} + 2^{11}$，求 p 的值。
If $p = 2 - 2^2 - 2^3 - 2^4 - \ldots - 2^9 - 2^{10} + 2^{11}$, find the value of p .

$p =$

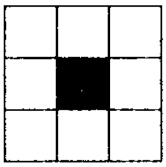
2. 已知 x、y、z 為 3 個相異實數。若 $x+\dfrac{1}{y}=y+\dfrac{1}{z}=z+\dfrac{1}{x}$ 及 $m = x^2y^2z^2$。
求 m 的值。
Given that x, y, z are three distinct real numbers.
If $x+\dfrac{1}{y}=y+\dfrac{1}{z}=z+\dfrac{1}{x}$ and $m = x^2y^2 z^2$, find the value of m .

$m =$

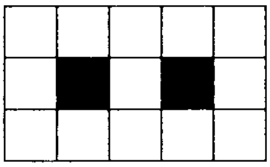
3. 已知 x 為一正實數，且滿足 $x\cdot 3^x = 3^{18}$。若 k 是一正整數且 $k < x < k + 1$，
求 k 的值。
Given that x is a positive real number and $x\cdot 3^x = 3^{18}$.
If k is a positive integer and $k < x < k + 1$, find the value of k .

$k =$

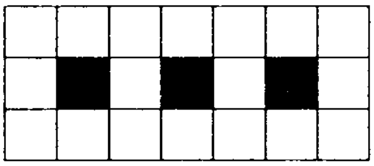
4. 圖一所示為利用黑白兩種顏色湊成有規律的圖形。
求第 95 個圖形的白色格子的數目 。
Figure 1 shows the sequence of figures that are made of squares of white and black. Find the number of white squares in the 95^{th} figure.



第一個
1st figure



第二個
2nd figure



第三個
3rd figure

圖一

Figure 1

FOR OFFICIAL USE

Score for accuracy		× Mult. factor for speed		=	Team No.	
		+ Bonus score			Time	
		Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

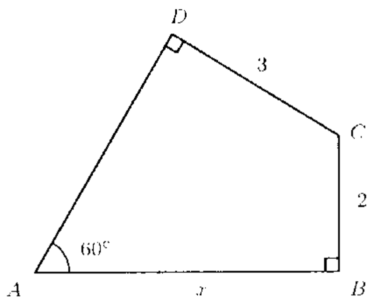
1. 求 $101^{303}+301^{101}$ 的最小質因子。
   Find the smallest prime factor of $101^{303}+301^{101}$.

2. 設 n 為 $\dfrac{1}{\frac{1}{1980}+\frac{1}{1981}+\cdots+\frac{1}{2009}}$ 的整數部分，求 n 的值。
   Let n be the integral part of $\dfrac{1}{\frac{1}{1980}+\frac{1}{1981}+\cdots+\frac{1}{2009}}$. Find the value of n.

   $n =$

3. 在圖一中，若 $\angle A=60°$，$\angle B=\angle D=90°$。$BC=2$，$CD=3$ 及 $AB=x$，求 x 的值。
   In Figure 1, $\angle A=60°$, $\angle B=\angle D=90°$. $BC=2$, $CD=3$ and $AB=x$, find the value of x.

   $x =$



圖一
Figure 1

4. 已知函數 f 對所有實數 x 皆滿足 $f(2+x)=f(2-x)$，且 $f(x)=0$ 恰好有四個相異實根。求這四個相異實根之和。
   Given that the function f satisfies $f(2+x)=f(2-x)$ for every real number x and that $f(x)=0$ has exactly four distinct real roots. Find the sum of these four distinct real roots.

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 a 為整數及 $a \neq 1$。已知方程 $(a-1)x^2 - mx + a = 0$ 的兩根均為正整數。求 m 的值。
Let a be an integer and $a \neq 1$. Given that the equation $(a-1)x^2 - mx + a = 0$ has two roots which are positive integers. Find the value of m.

$m =$

2. 已知 x 為一實數及 $y=\sqrt{x^2-2x+2}+\sqrt{x^2-10x+34}$。求 y 的最小值。

Given that x is a real number and $y=\sqrt{x^2-2x+2}+\sqrt{x^2-10x+34}$.

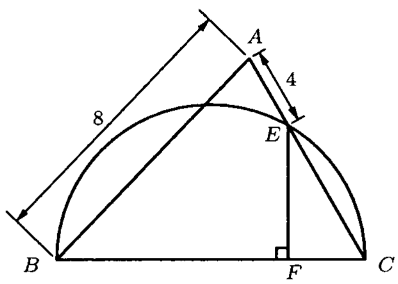
Find the minimum value of y.

$y =$

3. 已知 A、B、C 為正整數，且 A、B 和 C 的最大公因數等於 1。若 A、B、C 滿足 $A\log_{500}5 + B\log_{500}2 = C$，求 $A+B+C$ 的值。
Given that A, B, C are positive integers with their greatest common divisor equal to 1. If A, B, C satisfy $A\log_{500}5 + B\log_{500}2 = C$, find the value of $A+B+C$.

$A+B+C=$

4. 在圖一中，BEC 是一半圓形及 F 是直徑 BC 上的一點。已知 $BF : FC = 3 : 1$，$AB = 8$ 及 $AE = 4$。求 EC 的長度。
In figure 1, BEC is a semicircle and F is a point on the diameter BC. Given that $BF : FC = 3 : 1$, $AB = 8$ and $AE = 4$. Find the length of EC.



圖一
Figure 1

$EC =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2009 – 2010)
Final Event Spare (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 n 為一正整數。若 $n^2 + 5n + 13$ 為一完全平方數，求 n 的值。
Given that n is a positive integer.
If $n^2 + 5n + 13$ is a perfect square, find the value of n.

$n =$

2. 已知 $1^3 + 2^3 + \ldots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$，求 $11^3 + 12^3 + \ldots + 24^3$ 的值。
Given that $1^3 + 2^3 + \ldots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$. Find the value of $11^3 + 12^3 + \ldots + 24^3$.

3. 若 P 是等邊三角形 ABC 內部的隨意一點，求ΔABP 的面積同時大於ΔACP 及ΔBCP 的面積的概率。
If P is an arbitrary point in the interior of the equilateral triangle ABC, find the probability that the area of ΔABP is greater than each of the areas of ΔACP and ΔBCP.

4. 共有多少個正整數 m 使得通過點 $A(-m, 0)$ 及點 $B(0, 2)$的直綫亦通過 $P(7, k)$，其中 k 為一正整數？
How many positive integers m are there for which the straight line passing through points $A(-m, 0)$ and $B(0, 2)$ and also passes through the point $P(7, k)$, where k is a positive integer?

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
			Total score					Min.	Sec.