Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event Sample (Individual)

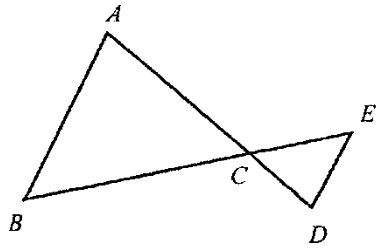
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 a、b、c 及 d 為方程 $x^4 - 15x^2 + 56 = 0$ 相異的根。
若 $R = a^2 + b^2 + c^2 + d^2$，求 R 的值。
Let a, b, c and d be the distinct roots of the equation $x^4 - 15x^2 + 56 = 0$.
If $R = a^2 + b^2 + c^2 + d^2$, find the value of R.

$R =$

2. 如圖一，AD 及 BE 為直綫且 $AB = AC$ 及 $AB \parallel ED$。
若 $\angle ABC = R°$及$\angle ADE = S°$，求 S 的值。
In Figure 1, AD and BE are straight lines with $AB = AC$ and $AB \parallel ED$.
If $\angle ABC = R°$ and $\angle ADE = S°$, find the value of S.

$S =$



圖一
Figure 1

3. 設 $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^S$ 及 $T = \sqrt{\dfrac{\log(1+F)}{\log 2}}$，求 T 的值。

Let $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^S$ and $T = \sqrt{\dfrac{\log(1+F)}{\log 2}}$, find the value of T.

$T =$

4. 設 f(x)是一個函數使得對所有整數 $n \ge 6$ 時，$f(n) = (n-1)\,f(n-1)$及 $f(n) \ne 0$。
若 $U = \dfrac{f(T)}{(T-1)f(T-3)}$，求 U 的值。
Let f(x) be a function such that $f(n) = (n-1)\,f(n-1)$
and $f(n) \ne 0$ hold for all integers $n \ge 6$. If $U = \dfrac{f(T)}{(T-1)f(T-3)}$, find the value of U.

$U =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 a、b 及 c 的平均值為 12，和 $2a+1$、$2b+2$、$2c+3$ 及 2 的平均值為 P，求 P 的值。
If the average of a, b and c is 12, and the average of $2a+1$, $2b+2$, $2c+3$ and 2 is P, find the value of P.

$P =$

2. 設 $20112011 = aP^5 + bP^4 + cP^3 + dP^2 + eP + f$，其中 a、b、c、d、e 及 f 為整數及 $0 \le a, b, c, d, e, f < P$。若 $Q = a + b + c + d + e + f$，求 Q 的值。
Let $20112011 = aP^5 + bP^4 + cP^3 + dP^2 + eP + f$, where a, b, c, d, e and f are integers and $0 \le a, b, c, d, e, f < P$. If $Q = a + b + c + d + e + f$, find the value of Q.

$Q =$

3. 若 R 為 $8^Q + 7^{10Q} + 6^{100Q} + 5^{1000Q}$ 的個位數，求 R 的值。
If R is the units digit of the value of $8^Q + 7^{10Q} + 6^{100Q} + 5^{1000Q}$, find the value of R.

$R =$

4. 若 S 為安排 R 個人圍成圓形的數目，求 S 的值。
If S is the number of ways to arrange R persons in a circle, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
			Total score					Min.	Sec.

Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若方程組 $\begin{cases} x+y=P \\ 3x+5y=13 \end{cases}$ 的解為正整數，求 P 的值。

   If the solution of the system of equations $\begin{cases} x+y=P \\ 3x+5y=13 \end{cases}$ are positive integers, find the value of P.

   $P =$

2. 若 $x+y=P$，$x^2+y^2=Q$ 及 $x^3+y^3=P^2$，求 Q 的值。

   If $x+y=P, x^2+y^2=Q$ and $x^3+y^3=P^2$, find the value of Q.

   $Q =$

3. 若 a 及 b 為相異質數且 $a^2-aQ+R=0$ 及 $b^2-bQ+R=0$，求 R 的值。

   If a and b are distinct prime numbers and $a^2-aQ+R=0$ and $b^2-bQ+R=0$, find the value of R.

   $R =$

4. 若 $S>0$ 及 $\frac{1}{S(S-1)}+\frac{1}{(S+1)S}+\cdots+\frac{1}{(S+20)(S+19)}=1-\frac{1}{R}$，求 S 的值。

   If $S>0$ and $\frac{1}{S(S-1)}+\frac{1}{(S+1)S}+\cdots+\frac{1}{(S+20)(S+19)}=1-\frac{1}{R}$, find the value of S.

   $S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 P 為一質數，而且方程 $x^2 + 2(P+1)x + P^2 - P - 14 = 0$ 的根為整數，求 P 的最小值。
If P is a prime number and the roots of the equation $x^2 + 2(P+1)x + P^2 - P - 14 = 0$ are integers, find the least value of P.

$P =$

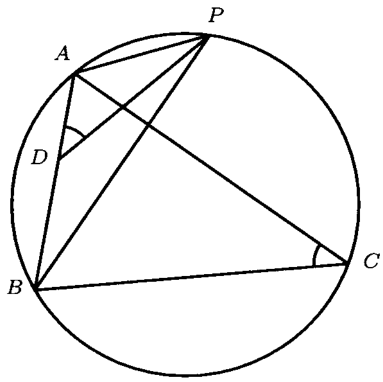
2. 已知 $x^2 + ax + b$ 為 $2x^3 + 5x^2 + 24x + 11$ 及 $x^3 + Px - 22$ 的公因式。若 $Q = a + b$，求 Q 的值。
Given that $x^2 + ax + b$ is a common factor of $2x^3 + 5x^2 + 24x + 11$ and $x^3 + Px - 22$. If $Q = a + b$, find the value of Q.

$Q =$

3. 若 R 為一正整數及 $R^3 + 4R^2 + (Q - 93)R + 14Q + 10$ 為一質數，求 R 的值。
If R is a positive integer and $R^3 + 4R^2 + (Q - 93)R + 14Q + 10$ is a prime number, find the value of R.

$R =$

4. 在圖一中，AP、AB、PB、PD、AC 及 BC 為綫段及 D 為 AB 上的一點。若 AB 的長度為 AD 的長度的 R 倍，$\angle ADP = \angle ACB$ 及 $S = \dfrac{PB}{PD}$，求 S 的值。
In Figure 1, AP, AB, PB, PD, AC and BC are line segments and D is a point on AB. If the length of AB is R times that of AD, $\angle ADP = \angle ACB$ and $S = \dfrac{PB}{PD}$, find the value of S.



圖一
Figure 1

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 考慮函數 $y = \sin x + \sqrt{3}\cos x$。設 a 為 y 的最大值。求 a 的值。
Consider the function $y = \sin x + \sqrt{3}\cos x$. Let a be the maximum value of y. Find the value of a.

$a =$

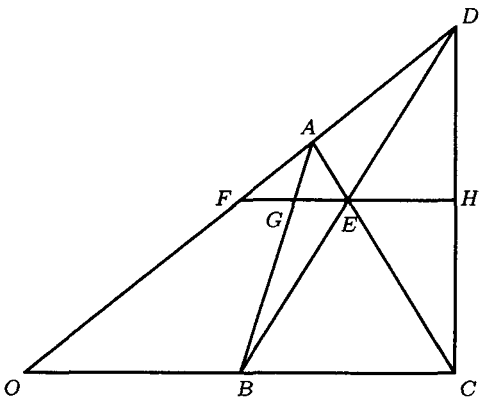
2. 若 b 及 y 滿足 $|b-y| = b + y - a$ 及 $|b+y| = b + a$。求 b 的值。
Find the value of b if b and y satisfy $|b-y| = b + y - a$ and $|b+y| = b + a$.

$b =$

3. 設 x、y 及 z 為正整數。若 $|x-y|^{2010} + |z-x|^{2011} = b$，而且 $c = |x-y| + |y-z| + |z-x|$，求 c 的值。
Let x, y and z be positive integers. If $|x-y|^{2010} + |z-x|^{2011} = b$ and $c = |x-y| + |y-z| + |z-x|$, find the value of c.

$c =$

4. 在圖一中，ODC 為一三角形。已知 FH、AB、AC 及 BD 為綫段使得 AB 及 FH 相交於 G，綫段 AC、BD 及 FH 相交於 E，$GE = 1$，$EH = c$ 及 $FH // OC$。若 $d = EF$，求 d 的值。
In Figure 1, let ODC be a triangle. Given that FH, AB, AC and BD are line segments such that AB intersects FH at G, AC, BD and FH intersect at E, $GE = 1$, $EH = c$ and $FH // OC$. If $d = EF$, find the value of d.



圖一
Figure 1

$d =$

FOR OFFICIAL USE

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		+ Bonus score				Time	
		Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event Spare (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 P 為邊長為整數小於或等於 9 的三角形的數目。求 P 的值。
Let P be the number of triangles whose side lengths are integers less than or equal to 9. Find the value of P.

$P =$

2. 設 $Q = \log_{128} 2^3 + \log_{128} 2^5 + \log_{128} 2^7 + \ldots + \log_{128} 2^P$。求 Q 的值。
Let $Q = \log_{128} 2^3 + \log_{128} 2^5 + \log_{128} 2^7 + \ldots + \log_{128} 2^P$. Find the value of Q.

$Q =$

3. 考慮直綫 $12x - 4y + (Q - 305) = 0$。
若 x-軸、y-軸及此直綫所形成的三角形的面積為 R 平方單位，求 R 的值。
Consider the line $12x - 4y + (Q - 305) = 0$. If the area of the triangle formed by the x-axis, the y-axis and this line is R square units, what is the value of R?

$R =$

4. 若 $x + \frac{1}{x} = R$ 及 $x^3 + \frac{1}{x^3} = S$，求 S 的值。
If $x + \frac{1}{x} = R$ and $x^3 + \frac{1}{x^3} = S$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event Sample (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知三角形三邊的長度分別是 a cm、2 cm 及 b cm，其中 a 和 b 是整數且 $a \le 2 \le b$。若有 q 種不全等的三角形滿足上述條件，求 q 的值。
Given some triangles with side lengths a cm, 2 cm and b cm, where a and b are integers and $a \le 2 \le b$. If there are q non-congruent classes of triangles satisfying the above conditions, find the value of q.

$q =$

2. 已知方程 $|x| - \frac{4}{x} = \frac{3|x|}{x}$ 有 k 個相異實根，求 k 的值。
Given that the equation $|x| - \frac{4}{x} = \frac{3|x|}{x}$ has k distinct real root(s), find the value of k.

$k =$

3. 已知 x 及 y 為非零實數且滿足方程 $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ 及 $x - y = 7$。若 $w = x + y$，求 w 的值。
Given that x and y are non-zero real numbers satisfying the equations $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ and $x - y = 7$. If $w = x + y$, find the value of w.

$w =$

4. 已知 x 及 y 為實數且 $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$。設 $p = |x| + |y|$，求 p 的值。
Given that x and y are real numbers and $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$.
Let $p = |x| + |y|$, find the value of p.

$p =$

FOR OFFICIAL USE

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				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event 1 (Group)

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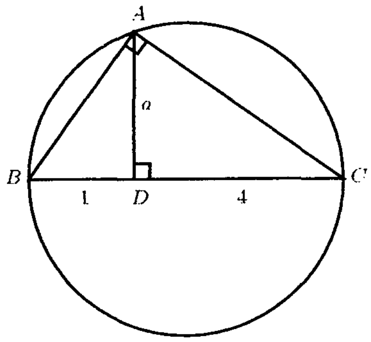
1. 在圖一中，BC 為圓的直徑，A 為圓上的一點，AB、AC 及 AD 為綫段，而且 AD 垂直 BC。若 $BD = 1$，$DC = 4$ 及 $AD = a$，求 a 的值。
In Figure 1, BC is the diameter of the circle. A is a point on the circle, AB and AC are line segments and AD is a line segment perpendicular to BC.
If $BD = 1$, $DC = 4$ and $AD = a$, find the value of a.



圖一
Figure 1

$a =$

2. 若 $b = 1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{-\frac{1}{2}}}}}$，求 b 的值。

If $b = 1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{1 - \cfrac{1}{-\frac{1}{2}}}}}$, find the value of b.

$b =$

3. 若 x、y 及 z 為實數，$xyz \neq 0$，$2xy = 3yz = 5xz$ 及 $c = \frac{x+3y-3z}{x+3y-6z}$。求 c 的值。

If x, y and z are real numbers, $xyz \neq 0$, $2xy = 3yz = 5xz$ and $c = \frac{x+3y-3z}{x+3y-6z}$, find the value of c.

$c =$

4. 若 x 為一整數滿足 $\log_{1/4}(2x+1) < \log_{1/2}(x-1)$，求 x 的最大值。
If x is an integer satisfying $\log_{1/4}(2x+1) < \log_{1/2}(x-1)$, find the maximum value of x.

$x =$

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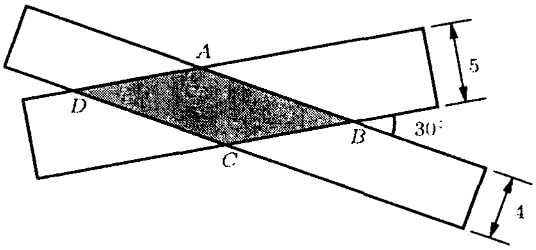
除非特別聲明，答案須用數字表達，並化至最簡。

1. 在圖一中，兩闊度為 4 及 5 單位的長方形間的夾角為 30°。
求重疊部份的面積。
In Figure 1, two rectangles with widths 4 and 5 units cross each other at 30°. Find the area of the overlapped region.

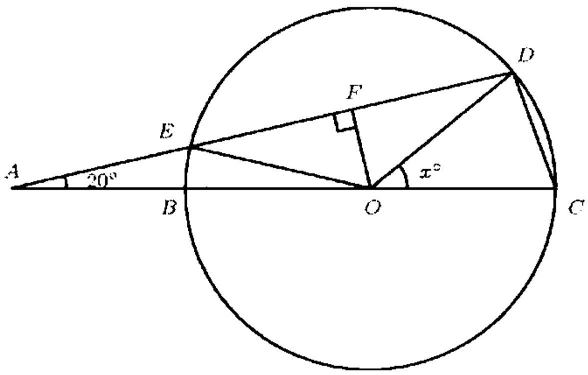


圖一
Figure 1

2. 從 1 到 100 選取兩整數(容許重覆)其和大於 100。問可選得多少對？
From 1 to 100, take a pair of integers (repetitions allowed) so that their sum is greater than 100. How many ways are there to pick such pairs?

3. 在圖二中的圓，其圓心為 O 及半徑為 r，三角形 ACD 與圓相交於 B、C、D 及 E 點。綫段 AE 的長度與圓的半徑相同。
若 $\angle DAC = 20°$ 及 $\angle DOC = x°$，
求 x 的值。
In Figure 2, there is a circle with centre O and radius r. Triangle ACD intersects the circle at B, C, D and E. Line segment AE has the same length as the radius. If $\angle DAC = 20°$ and $\angle DOC = x°$, find the value of x.

$x =$



圖二
Figure 2

4. 已知 $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=0$ 及 $\frac{1}{x}-\frac{6}{y}-\frac{5}{z}=0$。若 $P=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$，求 P 的值。
Given that $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=0$ and $\frac{1}{x}-\frac{6}{y}-\frac{5}{z}=0$. If $P=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$, find the value of P.

$P =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 a 為一正整數及 $a^2 + 100a$ 為一質數，求 a 的最大值。
If a is a positive integer and $a^2 + 100a$ is a prime number, find the maximum value of a.

$a =$

2. 設 a、b 及 c 為實數。若 1 為 $x^2 + ax + 2 = 0$ 的根及 a 和 b 為 $x^2 + 5x + c = 0$ 的根，求 $a + b + c$ 的值。
Let a, b and c be real numbers. If 1 is a root of $x^2 + ax + 2 = 0$ and a and b be roots of $x^2 + 5x + c = 0$, find the value of $a + b + c$.

$a+b+c =$

3. 設 x 及 y 為正實數且 $x<y$。若 $\sqrt{x}+\sqrt{y}=1$、$\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3}$ 及 $x<y$，求 $y-x$ 的值。
Let x and y be positive real numbers with $x<y$.
If $\sqrt{x}+\sqrt{y}=1$, $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3}$ and $x<y$, find the value of $y-x$.

$y-x =$

4. 把數字 1, 2, … , 10 分成兩組並設 P_1 及 P_2 分別為該兩組的乘積。
若 P_1 為 P_2 的倍數，求 $\frac{P_1}{P_2}$ 的最小值。
Spilt the numbers 1, 2, … , 10 into two groups and let P_1 be the product of the first group and P_2 the product of the second group.
If P_1 is a multiple of P_2, find the minimum value of $\frac{P_1}{P_2}$.

$\frac{P_1}{P_2} =$

FOR OFFICIAL USE

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Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $P=2\sqrt[4]{2007\cdot 2009\cdot 2011\cdot 2013+10\cdot 2010\cdot 2010-9}-4000$，求 P 的值。
If $P=2\sqrt[4]{2007\cdot 2009\cdot 2011\cdot 2013+10\cdot 2010\cdot 2010-9}-4000$, find the value of P.

$P=$

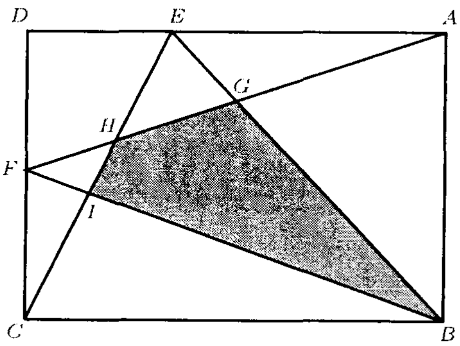
2. 若 $9x^2+nx+1$ 及 $4y^2+12y+m$ 為平方數及 $n>0$，求 $\frac{n}{m}$ 的值。
If $9x^2+nx+1$ and $4y^2+12y+m$ are squares with $n>0$, find the value of $\frac{n}{m}$.

$\frac{n}{m}=$

3. 設 n 及 $\frac{47}{5}\left(\frac{4}{47}+\frac{n}{141}\right)$ 為正整數。若 r 為 n 被 15 除的餘數，求 r 的最值。
Let n and $\frac{47}{5}\left(\frac{4}{47}+\frac{n}{141}\right)$ be positive integers. If r is the remainder of n divided by 15, find the value of r.

$r=$

4. 在圖一中，$ABCD$ 為一長方形，及 E 及 F 分別為綫段 AD 及 DC 上的點。點 G 為綫段 AF 及 BE 的交點，點 H 為綫段 AF 及 CE 的交點，點 I 為綫段 BF 及 CE 的交點。若 AGE，$DEHF$ 及 CIF 的面積分別為 2、3 及 1，求灰色部份 $BGHI$ 的面積。



圖一 Figure 1

In figure 1, $ABCD$ is a rectangle, and E and F are points on AD and DC, respectively. Also, G is the intersection of AF and BE, H is the intersection of AF and CE, and I is the intersection of BF and CE.
If the areas of AGE, $DEHF$ and CIF are 2, 3 and 1, respectively, find the area of the grey region $BGHI$.

Shaded area =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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Hong Kong Mathematics Olympiad (2010 – 2011)
Final Event Spare (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 α 及 β 為方程 $y^2 - 6y + 5 = 0$ 的實根。設 m 為 $|x-\alpha| + |x-\beta|$ 對任何實數 x 的最小值。求 m 的值。
Let α and β be the real roots of $y^2 - 6y + 5 = 0$.
Let m be the minimum value of $|x-\alpha| + |x-\beta|$ over all real values of x.
Find the value of m.
$m =$

2. 設 α、β、γ 為實數且滿足 $\alpha + \beta + \gamma = 2$ 及 $\alpha\beta\gamma = 4$。設 v 為 $|\alpha| + |\beta| + |\gamma|$ 的最小值，求 v 的值。
Let α, β, γ be real numbers satisfying $\alpha + \beta + \gamma = 2$ and $\alpha\beta\gamma = 4$.
Let v be the minimum value of $|\alpha| + |\beta| + |\gamma|$. Find the value of v.
$v =$

3. 設 $y = |x+1| - 2|x| + |x-2|$ 及 $-1 \le x \le 2$。設 α 為 y 的最大值，求 α 的值。
Let $y = |x+1| - 2|x| + |x-2|$ and $-1 \le x \le 2$.
Let α be the maximum value of y. Find the value of α.
$\alpha =$

4. 設 F 為方程 $x^2 + y^2 + z^2 + w^2 = 3(x + y + z + w)$ 的整數解的數目。求 F 的值。
Let F be the number of integral solutions of $x^2 + y^2 + z^2 + w^2 = 3(x + y + z + w)$.
Find the value of F.
$F =$

FOR OFFICIAL USE

Score for accuracy $\times$ Mult. factor for speed $=$ Team No.

$+$ Bonus score Time

Total score Min. Sec.