

Hong Kong Mathematics Olympiad 2010-2011
Heat Event (Individual)

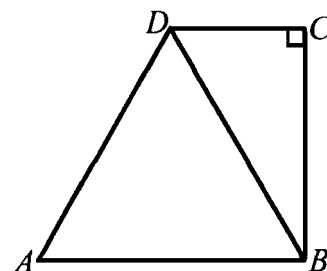
除非特別聲明，答案須用數字表達，並化至最簡。

時限：40 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 求 2^{2011} 除以 13 的餘數。
Find the remainder when 2^{2011} is divided by 13.
2. 已知 $x^2 + y^2 = 1$ ，求 $2x + 5y^2$ 的極大值。
Given that $x^2 + y^2 = 1$, find the maximum value of $2x + 5y^2$.
3. 已知 $a + b = \sqrt{\sqrt{2011} + \sqrt{2010}}$ 及 $a - b = \sqrt{\sqrt{2011} - \sqrt{2010}}$ ，求 ab 的值。(答案以根式表示)
Given that $a + b = \sqrt{\sqrt{2011} + \sqrt{2010}}$ and $a - b = \sqrt{\sqrt{2011} - \sqrt{2010}}$, find the value of ab .
(Give your answer in surd form)
4. 在 $\triangle ABC$ 內，分別垂直於三條邊 AB 、 BC 及 CA 的高的比是 3:4:5。若三條邊的長均為整數，求 AB 的最小值。
In $\triangle ABC$, the ratio of the altitudes perpendicular to the three sides AB , BC and CA is 3 : 4 : 5. If the lengths of the three sides are integers, find the minimum value of AB .
5. 整數 x 減去 12 後是一個整數的平方。將 x 加上 19 後則是另一個整數的平方。求 x 的值。
An integer x minus 12 is the square of an integer. x plus 19 is the square of another integer. Find the value of x .
6. 甲、乙及丙三人互相傳球。甲首先將球傳出。
有多少不同方案使得經過 5 次傳球後，球會回傳給甲？
 A , B and C pass a ball among themselves. A is the first one to pass the ball to other one.
In how many ways will the ball be passed back to A after 5 passes?
7. 求 $\sqrt{7 - \sqrt{12} - \sqrt{13 - 2\sqrt{12}}}$ 的值。
Find the value of $\sqrt{7 - \sqrt{12} - \sqrt{13 - 2\sqrt{12}}}$.
8. 學校推出每張面值為 \$10、\$15、\$25 及 \$40 的四種賣物券。甲班用若干張 \$100 紙幣買了 30 張賣物券，包括其中兩種賣物券各 5 張及另外兩種賣物券各 10 張。問甲班共用了多少張 \$100 紙幣購買賣物券？
A school issues 4 types of raffle tickets with face values \$10, \$15, \$25 and \$40.
Class A uses several one-hundred dollar notes to buy 30 raffle tickets, including 5 tickets each for two of the types and 10 tickets each for the other two types.
How many one-hundred dollars notes Class A use to buy the raffle tickets?
9. 某長方形的長和闊均為整數。若面積比周界大 9，求周界的值。
The length and the width of a rectangle are integers.
If its area is larger than its perimeter by 9, find the perimeter.
10. 如圖， $ABCD$ 為個梯形，其中 $\angle C = 90^\circ$ 。
若等邊三角形 ABD 的面積為 $16\sqrt{3}$ ，求 梯形 $ABCD$ 的面積。
In the figure, $ABCD$ is a trapezium with $\angle C = 90^\circ$.
If the area of the equilateral triangle ABD is $16\sqrt{3}$,
find the area of trapezium $ABCD$.



*** 試卷完 End of Paper ***

Hong Kong Mathematics Olympiad 2010-2011

Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。

時限：20 分鐘

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 若 $(1000-a)(1000-b)(1000-c)(1000-d)(1000-e) = 24^2$ ，其中 a, b, c, d 及 e 為偶數，且 $a > b > c > d > e$ ，求 a, b, c, d 及 e 的值。
If $(1000-a)(1000-b)(1000-c)(1000-d)(1000-e) = 24^2$, where a, b, c, d and e are even numbers and $a > b > c > d > e$, find the values of a, b, c, d and e .

2. 以 \overline{ab} 表示一個兩位數，其十位是 a ，個位是 b ，且 $R_{\overline{ab}}$ 表示 \overline{ab} 除以 $a+b$ 的餘數。求 $R_{\overline{ab}}$ 的最大值。
 \overline{ab} denotes a two digit number with a as tens digit and b as the unit digit. $R_{\overline{ab}}$ is the remainder when \overline{ab} is divided by $a+b$. Find the maximum value of $R_{\overline{ab}}$.

3. 已知 a, b, c 為整數，且 $a+b=2011$ ， $c-a=2010$ ， $a < b$ 。求 $a+b+c$ 的可能最大值。
Given that a, b and c are integers, and $a+b=2011$, $c-a=2010$, $a < b$. Find the greatest possible value of $a+b+c$.

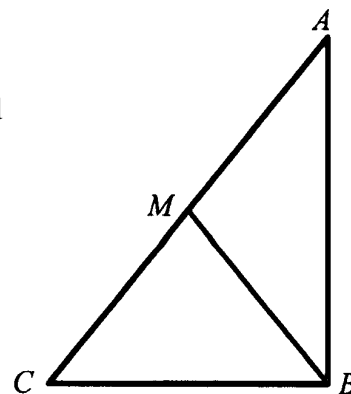
4. 已知 n 為一正整數，且 $n^4 - 18n^2 + 49$ 為一質數。求 n 的值。
Given that n is a positive integer and $n^4 - 18n^2 + 49$ is a prime number, find the value of n .

5. 已知 $f(x) = \frac{4^x}{4^x + 2}$ ，其中 x 是實數。
求 $f\left(\frac{1}{2011}\right) + f\left(\frac{2}{2011}\right) + f\left(\frac{3}{2011}\right) + \cdots + f\left(\frac{2009}{2011}\right) + f\left(\frac{2010}{2011}\right)$ 的值。

Given that $f(x) = \frac{4^x}{4^x + 2}$, where x is a real number, find the value of

$$f\left(\frac{1}{2011}\right) + f\left(\frac{2}{2011}\right) + f\left(\frac{3}{2011}\right) + \cdots + f\left(\frac{2009}{2011}\right) + f\left(\frac{2010}{2011}\right).$$

6. 如下圖， M 為 AC 上的一點，且 $AM = MC = BM = 3$ 。
求 $AB + BC$ 的最大值。
In the figure below, M is a point on AC , $AM = MC = BM = 3$. Find the maximum value of $AB + BC$.

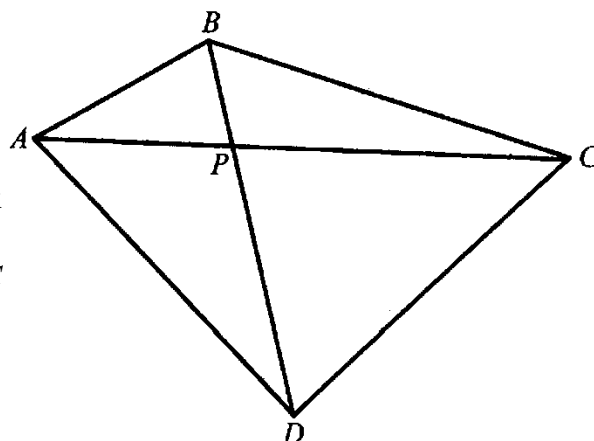


7. 已知 $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$ 且 $\frac{2011!}{10^k}$ 是整數，其中 k 是正整數。
若 S 是 k 的所有可能值之和，求 S 的值。
Given that $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$ and $\frac{2011!}{10^k}$ is an integer, where k is a positive integer. If S is the sum of all possible values of k , find the value of S .

8. 已知 a 、 b 、 c 及 d 為非負整數，且 $ac + bd + ad + bc = 2011$ 。求 $a + b + c + d$ 的值。
 Given that a, b, c and d are non-negative integers and $ac + bd + ad + bc = 2011$.
 Find the value of $a + b + c + d$.

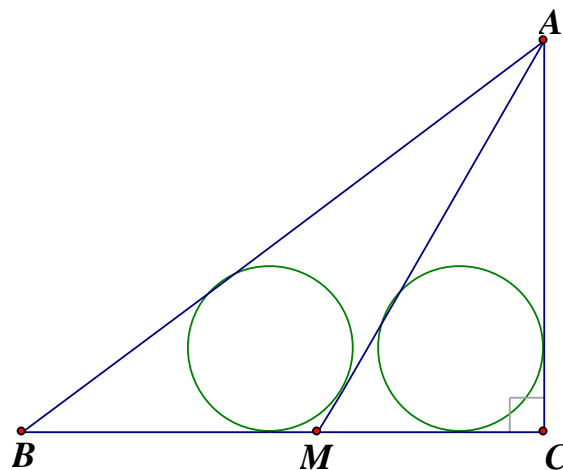
9. 如圖， $ABCD$ 為一凸四邊形， $\angle BAC = 27^\circ$ ， $\angle BCA = 18^\circ$ ， $\angle BDC = 54^\circ$ ， $\angle BDA = 36^\circ$ ，且四邊形的對角線 AC 、 BD 相交於 P 。求 $\angle CPB$ 。

As shown in the figure, $ABCD$ is a convex quadrilateral, $\angle BAC = 27^\circ$, $\angle BCA = 18^\circ$, $\angle BDC = 54^\circ$, $\angle BDA = 36^\circ$. The diagonals AC and BD intersect at P . Find $\angle CPB$.



10. 如圖， $AC = 3$ ， $BC = 4$ 及 $\angle C = 90^\circ$ 。M 是 BC 上的一點使得 $\triangle ABM$ 及 $\triangle ACM$ 的內切圓的半徑相等。求 AM 的長。

As shown in the figure, $AC = 3$, $BC = 4$ and $\angle C = 90^\circ$. M is a point on BC such that the radii of the incircles in $\triangle ABM$ and $\triangle ACM$ are equal. Find the length of AM .



Hong Kong Mathematics Olympiad 2010 – 2011
Heat Event (Geometric Construction)
香港數學競賽 2010 – 2011
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。

時限：20 分鐘

All working (including geometric drawing) must be clearly shown.

此部份滿分為十分。The full marks of this part is 10 marks.

Time allowed: 20 minutes

School Code: _____

School Name: _____

第一題 Question No. 1

已知一直線 L ，及兩點 P 、 Q 位於 L 的同一方。試在 L 上作一點 T 使得 PT 及 QT 的長度之和最小。(提示：可考慮 P 點於直線 L 上作反射的影像)

Given a straight line L , and two points P and Q lying on the same side of L . Mark a point T on L so that the sum of the lengths of PT and QT is minimal. (Hint: Consider the reflection image of P about the line L .)



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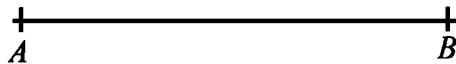
School Code: _____

School Name: _____

第二題 Question No. 2

如圖所示為一線段 AB 。 D 為一固定點，且 A 、 B 、 D 不共線。試作 $\triangle ABC$ ，使得 C 、 B 及 D 共線，及其餘兩條邊的長度差(即 $AC - BC$)等如 BD 。

The figure shows a line segment AB . D is a fixed point such that A , B , D are not collinear. Construct a triangle ABC so that C , B and D are collinear and the difference between the other two sides of $\triangle ABC$ (i.e. $AC - BC$) is equal to BD .



$\times D$

圖二 Figure 2

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香港數學競賽 2010 – 2011

初賽(幾何作圖)

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School Code: _____

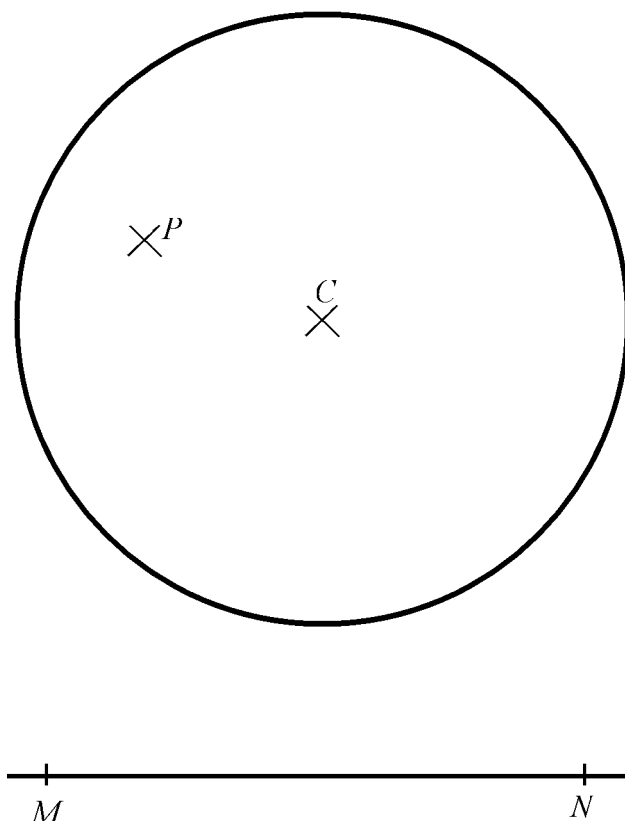
School Name: _____

第三題 Question No. 3

圖三所示為一以 C 為圓心的圓及一線段 MN 。 P 為該圓內的一點。試構作一通過 P 的弦 QR ，其中 Q 及 R 為圓周上的點，且 QR 的長度與 MN 的長度相等。

Figure 3 shows a circle of centre C and a line segment MN . P is a point lies inside the circle.

Construct a chord QR with points Q and R on the circumference of the circle so that it passes through P and its length is equal to that of MN .



圖三 Figure 3

*** 試卷完 End of Paper ***