Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event Sample (Individual)

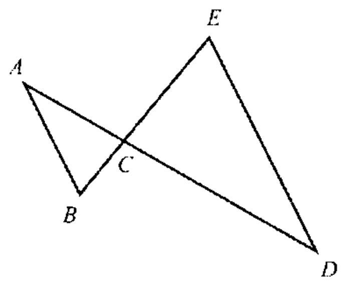
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 a、b、c 及 d 為方程 $x^4 - 15x^2 + 56 = 0$ 的根。
若 $P = a^2 + b^2 + c^2 + d^2$，求 P 的值。
Let a, b, c and d be the roots of the equation $x^4 - 15x^2 + 56 = 0$.
If $P = a^2 + b^2 + c^2 + d^2$, find the value of P.

$P =$

2. 如圖一，$AB = AC$ 及 $AB \mathbin{/\!/} ED$。若 $\angle ABC = P°$ 及 $\angle ADE = Q°$，求 Q 的值。
In Figure 1, $AB = AC$ and $AB \mathbin{/\!/} ED$. If $\angle ABC = P°$ and $\angle ADE = Q°$, find the value of Q.

$Q =$



圖一
Figure 1

3. 設 $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^Q$ 及 $R = \sqrt{\dfrac{\log(1+F)}{\log 2}}$，求 R 的值。

Let $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^Q$ and $R = \sqrt{\dfrac{\log(1+F)}{\log 2}}$, find the value of R.

$R =$

4. 設 f(x)是一個函數使得對所有正整數 n，$f(n) = (n-1)\,f(n-1)$ 及 $f(1) \neq 0$。
若 $S = \dfrac{f(R)}{(R-1)f(R-3)}$，求 S 的值。
Let f(x) be a function such that $f(n) = (n-1)\,f(n-1)$ and $f(1) \neq 0$ for all positive integers n. If $S = \dfrac{f(R)}{(R-1)f(R-3)}$, find the value of S.

$S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 A 是多項式 $x^4 + 6x^3 + 12x^2 + 9x + 2$ 的所有根的平方之和，求 A 的值。
If A is the sum of the squares of the roots of $x^4 + 6x^3 + 12x^2 + 9x + 2$, find the value of A.

$A =$

2. 設 x、y、z、w 為正 A 邊形的四個相連端點。若綫段 xy 的長度為 2 及四邊形 $xyzw$ 的面積是 $a+\sqrt{b}$，求 $B = 2^a \cdot 3^b$ 的值。
Let x, y, z, w be four consecutive vertices of a regular A-gon. If the length of the line segment xy is 2 and the area of the quadrilateral $xyzw$ is $a+\sqrt{b}$, find the value of $B = 2^a \cdot 3^b$.

$B =$

3. 若 C 是 B 的所有正因子之和，其中 B 的因子包括 1 和 B，求 C 的值。
If C is the sum of all positive factors of B, including 1 and B itself, find the value of C.

$C =$

4. 若 $C! = 10^D \cdot k$，其中 D 及 k 皆為整數且 k 不是 10 的倍數，求 D 的值。
If $C! = 10^D \cdot k$, where D and k are integers such that k is not divisible by 10, find the value of D.

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=			Team No.		
			+	Bonus score				Time		
				Total score					Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 P 是方程 $x^2+9x+13=2\sqrt{x^2+9x+21}$ 的所有實根之乘積，求 P 的值。
If the product of the real roots of the equation $x^2+9x+13=2\sqrt{x^2+9x+21}$ is P, find the value of P.

$P =$

2. 若 $\mathrm{f}(x)=\dfrac{25^x}{25^x+P}$ 及 $Q=\mathrm{f}\left(\dfrac{1}{25}\right)+\mathrm{f}\left(\dfrac{2}{25}\right)+\cdots+\mathrm{f}\left(\dfrac{24}{25}\right)$，求 Q 的值。
If $\mathrm{f}(x)=\dfrac{25^x}{25^x+P}$ and $Q=\mathrm{f}\left(\dfrac{1}{25}\right)+\mathrm{f}\left(\dfrac{2}{25}\right)+\cdots+\mathrm{f}\left(\dfrac{24}{25}\right)$, find the value of Q.

$Q =$

3. 若 $X=\sqrt{(100)(102)(103)(105)+(Q-3)}$ 是整數及 R 是 X 的個位數，求 R 的值。
If $X=\sqrt{(100)(102)(103)(105)+(Q-3)}$ is an integer and R is the units digit of X, find the value of R.

$R =$

4. 若 S 是方程 $\sqrt{2012}\cdot x^{\log_{2012}x}=x^R$ 的所有正根之乘積的最後 3 位數字(個位數，十位數，百位數)之和，求 S 的值。
If S is the sum of the last 3 digits (hundreds, tens, units) of the product of the positive roots of $\sqrt{2012}\cdot x^{\log_{2012}x}=x^R$, find the value of S.

$S =$

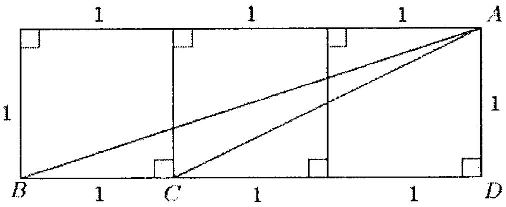
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 在圖一中，長方形由三個邊長為 1 之正方形組成。
若 $\alpha° = \angle ABD + \angle ACD$，求 α 的值。
In Figure 1, a rectangle is sub-divided into 3 identical squares of side length 1 .
If $\alpha° = \angle ABD + \angle ACD$,
find the value of α.



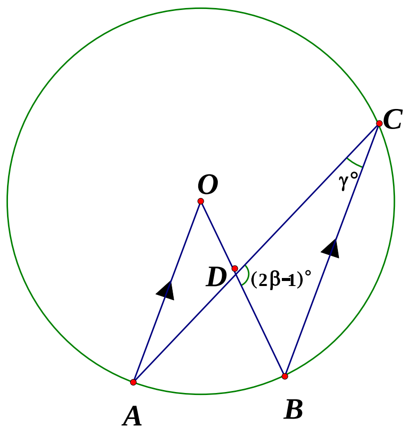
圖一 Figure 1

$\alpha =$

2. 設 ABC 為一銳角三角形。若 $\sin A = \dfrac{36}{\alpha}$，$\sin B = \dfrac{12}{13}$ 及 $\sin C = \dfrac{\beta}{y}$，求 β 的值，其中 β 及 y 是最簡化之代表形式。
Let ABC be an acute-angled triangle. If $\sin A = \dfrac{36}{\alpha}$, $\sin B = \dfrac{12}{13}$ and $\sin C = \dfrac{\beta}{y}$,
find the value of β, where β and y are in the lowest terms.

$\beta =$

3. 在圖二中，有一個圓心在 O 的圓，其圓周上有點 A、B 及 C，四條綫段：OA、OB、AC 與 BC，且 OA 與 BC 平行。
若 D 是 OB 及 AC 之交點且$\angle BDC = (2\beta - 1)°$ 及 $\angle ACB = \gamma°$，求 γ 的值。
In Figure 2, a circle at centre O has three points on its circumference, A, B and C. There are line segments OA, OB, AC and BC, where OA is parallel to BC. If D is the intersection of OB and AC with $\angle BDC = (2\beta - 1)°$ and $\angle ACB = \gamma°$,
find the value of γ.



圖二 Figure 2

$\gamma =$

4. 在 $(ax + b)^{2012}$ 的展開式中，a 與 b 為互質之正整數，若 x^{γ} 與 $x^{\gamma+1}$ 的系數相同，求 $\delta = a + b$ 的值。
In the expansion of $(ax + b)^{2012}$, where a and b are relatively prime positive integers. If the coefficients of x^{γ} and $x^{\gamma+1}$ are equal, find the value of $\delta = a + b$.

$\delta =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 A 為一正整數且 $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\cdots+\frac{1}{(A+1)(A+3)}=\frac{12}{25}$，求 A 的值。

If A is a positive integer such that $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\cdots+\frac{1}{(A+1)(A+3)}$,

find the value of A.

$A =$

2. 若 x 與 y 為正整數且 $x > y > 1$ 及 $xy = x + y + A$。設 $B=\frac{x}{y}$，求 B 的值。

If x and y be positive integers such that $x > y > 1$ and $xy = x + y + A$.

Let $B=\frac{x}{y}$, find the value of B.

$B =$

3. 設 f 為為一函數並滿足以下條件：
(i) 對所有正整數 n，$f(n)$ 必為整數；
(ii) $f(2) = 2$;
(iii) 對所有正整數 m 及 n，$f(mn) = f(m)\cdot f(n)$及
(iv) 當 $m > n$，$f(m) > f(n)$。
若 $C = f(B)$，求 C 的值。

Let f be a function satisfying the following conditions:
(i) $f(n)$ is an integer for every positive integer n;
(ii) $f(2) = 2$;
(iii) $f(mn) = f(m)\cdot f(n)$ for all positive integers m and n and
(iv) $f(m) > f(n)$ if $m > n$.
If $C = f(B)$, find the value of C.

$C =$

4. 設 D 為 2401×7^C (以十進制表示)的最後三位數字之和。求 D 的值。

Let D be the sum of the last three digits of 2401×7^C (in the denary system).
Find the value of D.

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.
			+	Bonus score			Time
				Total score			Min. / Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event Spare (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 P 為邊長為整數小於或等於 9 的三角形的數目。求 P 的值。
   Let P be the number of triangles whose side lengths are integers less than or equal to 9. Find the value of P.

   $P =$

2. 設 $Q = \log_{128} 2^3 + \log_{128} 2^5 + \log_{128} 2^7 + \ldots + \log_{128} 2^P$。求 Q 的值。
   Let $Q = \log_{128} 2^3 + \log_{128} 2^5 + \log_{128} 2^7 + \ldots + \log_{128} 2^P$. Find the value of Q.

   $Q =$

3. 考慮直綫 $12x - 4y + (Q - 305) = 0$。
   若 x-軸、y-軸及此直綫所形成的三角形的面積為 R 平方單位，求 R 的值。
   Consider the line $12x - 4y + (Q - 305) = 0$. If the area of the triangle formed by the x-axis, the y-axis and this line is R square units, what is the value of R?

   $R =$

4. 若 $x + \frac{1}{x} = R$ 及 $x^3 + \frac{1}{x^3} = S$，求 S 的值。
   If $x + \frac{1}{x} = R$ and $x^3 + \frac{1}{x^3} = S$, find the value of S.

   $S =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
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除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知三角形三邊的長度分別是 a cm、2 cm 及 b cm，其中 a 和 b 是整數且 $a \le 2 \le b$。若有 q 種不全等的三角形滿足上述條件，求 q 的值。
Given some triangles with side lengths a cm, 2 cm and b cm, where a and b are integers and $a \le 2 \le b$. If there are q non-congruent classes of triangles satisfying the above conditions, find the value of q.

$q =$

2. 已知方程 $|x| - \frac{4}{x} = \frac{3|x|}{x}$ 有 k 個相異實根，求 k 的值。
Given that the equation $|x| - \frac{4}{x} = \frac{3|x|}{x}$ has k distinct real root(s), find the value of k.

$k =$

3. 已知 x 及 y 為非零實數且滿足方程 $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ 及 $x - y = 7$。若 $w = x + y$，求 w 的值。
Given that x and y are non-zero real numbers satisfying the equations $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ and $x - y = 7$. If $w = x + y$, find the value of w.

$w =$

4. 已知 x 及 y 為實數且 $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$。設 $p = |x| + |y|$，求 p 的值。
Given that x and y are real numbers and $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$.
Let $p = |x| + |y|$, find the value of p.

$p =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
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				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event 1 (Group)

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1. 求 2011^{2011} 的十位數。
Calculate the tens digit of 2011^{2011}.

tens digit =

2. 設 a_1、a_2、a_3、⋯ 為一等差數列，公差是 1 及 $a_1 + a_2 + a_3 + \cdots + a_{100} = 2012$。如果 $P = a_2 + a_4 + a_6 + \cdots + a_{100}$，求 P 的值。
Let $a_1, a_2, a_3, \cdots$ be an arithmetic sequence with common difference 1 and $a_1 + a_2 + a_3 + \cdots + a_{100} = 2012$. If $P = a_2 + a_4 + a_6 + \cdots + a_{100}$, find the value of P.

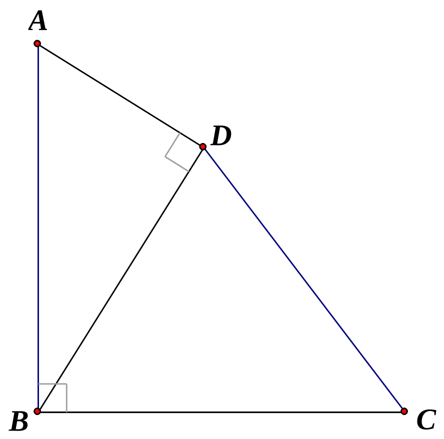
$P =$

3. 若 90! 可被 10^k 整除，當中 k 是正整數，求 k 的最大可能值。
If 90! is divisible by 10^k, where k is a positive integer, find the greatest possible value of k.

$k =$

4. 在圖一中，ΔABC 是一直角三形且 $AB \perp BC$。若 $AB = BC$，D 是一點使得 $AD \perp BD$，且 $AD = 5$ 及 $BD = 8$，求 ΔBCD 的面積的值。
In Figure 1, ΔABC is a right-angled triangle with $AB \perp BC$. If $AB = BC$, D is a point such that $AD \perp BD$ with $AD = 5$ and $BD = 8$, find the value of the area of ΔBCD.

$S_{\Delta BCD} =$



圖一 Figure 1

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 求 $2\times\tan 1°\times\tan 2°\times \tan 3°\times\ldots\times \tan 87°\times \tan 88°\times \tan 89°$的值。
Find the value of $2\times\tan 1°\times\tan 2°\times \tan 3°\times\ldots\times \tan 87°\times \tan 88°\times \tan 89°$.

2. 若方程$(x^2-3x+2)^2-3(x^2-3x)-4=0$ 有 K 個整數解，求 K 的值。
If there are K integers that satisfy the equation $(x^2-3x+2)^2-3(x^2-3x)-4=0$, find the value of K.

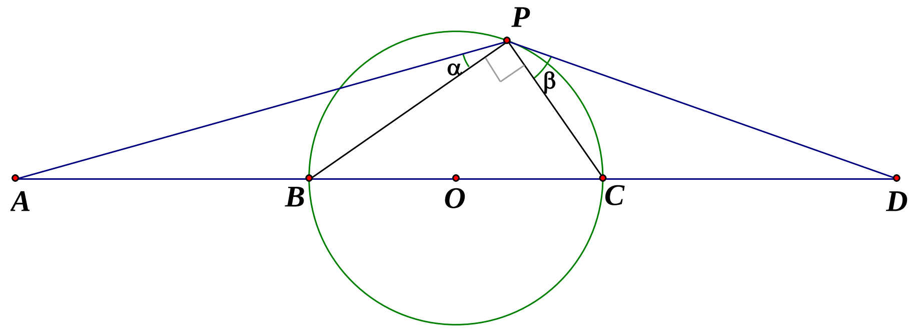
$K =$

3. 若 ℓ 為 $|x-2|+|x-47|$的最小值，求 ℓ 的值。
If ℓ is the minimum value of $|x-2|+|x-47|$, find the value of ℓ.

$\ell =$

4. 在圖一，圓有直徑 BC，圓心在 O，P、B 及 C 皆為圓周上的點。若 $AB=BC=CD$ 及 AD 為一綫段，$\alpha=\angle APB$ 及$\beta=\angle CPD$，求 $(\tan\alpha)(\tan\beta)$ 的值。
In Figure 1, P, B and C are points on a circle with centre O and diameter BC.
If $AB=BC=CD$ and AD is a line segment, $\alpha=\angle APB$ and $\beta=\angle CPD$, find the value of $(\tan\alpha)(\tan\beta)$.

$(\tan\alpha)(\tan\beta) =$



圖一 Figure 1

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)

Final Event 3 (Group)

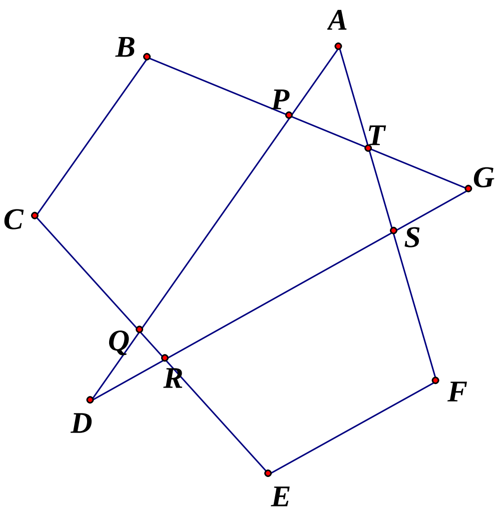
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $x=\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$，$y=\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$ 及 $192z=x^4+y^4+(x+y)^4$，求 z 的值。
Let $x=\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$, $y=\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$ and $192z=x^4+y^4+(x+y)^4$, find the value of z.

$z=$

2. 在圖一中，AD、DG、GB、BC、CE、EF 及 FA 都是直綫綫段。
若 $\angle FAD+\angle GBC+\angle BCE+\angle ADG+\angle CEF+\angle AFE+\angle DGB=r°$，求 r 的值。
In Figure 1, AD, DG, GB, BC, CE, EF and FA are line segments.
If $\angle FAD+\angle GBC+\angle BCE+\angle ADG+\angle CEF+\angle AFE+\angle DGB=r°$, find the value of r.

$r=$



圖一 Figure 1

3. 設 k 為正整數及函數 $f(k)$的定義是若 $\dfrac{k-1}{k}=0.k_1k_2k_3\ldots\ldots$，則 $f(k)=\overline{k_1k_2k_3}$，
例如 $f(3)=666$ 因為 $\dfrac{3-1}{3}=0.666\ldots\ldots$，求 $D=f(f(f(f(f(112)))))$的值。
Let k be positive integer and $f(k)$ a function that if $\dfrac{k-1}{k}=0.k_1k_2k_3\cdots\cdots$,
then $f(k)=\overline{k_1k_2k_3}$, for example, $f(3)=666$ because $\dfrac{3-1}{3}=0.666\cdots\cdots$,
find the value of $D=f(f(f(f(f(112)))))$.

$D=$

4. 若 F_n 為一整數值函數，其定義為 $F_n(k)=F_1(F_{n-1}(k))$，$n\ge 2$
且 $F_1(k)$是 k 的所有位數的平方之和，求 $F_{2012}(7)$的值。
If F_n is an integral valued function defined recursively by $F_n(k)=F_1(F_{n-1}(k))$ for $n\ge 2$ where $F_1(k)$ is the sum of squares of the digits of k, find the value of $F_{2012}(7)$.

$F_{2012}(7)=$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ] Team No. [ ]

\+ Bonus score [ ] Time [ ] [ ]

Total score [ ] Min. Sec.

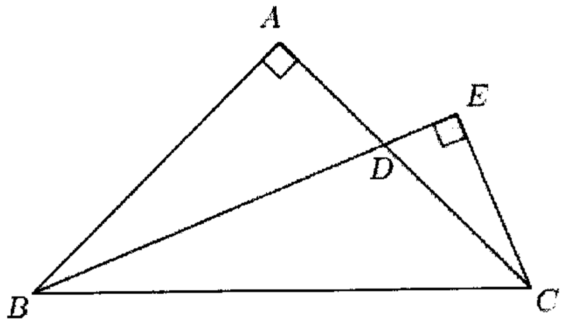
Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 在圖一中，ABC 及 EBC 是兩個直角三角形，$\angle BAC = \angle BEC = 90°$，$AB = AC$ 及 EDB 為 $\angle ABC$ 的角平分綫。求 $\dfrac{BD}{CE}$ 的值。

   In figure 1, ABC and EBC are two right-angled triangles, $\angle BAC = \angle BEC = 90°$，$AB = AC$ and EDB is the angle bisector of $\angle ABC$. Find the value of $\dfrac{BD}{CE}$.

   $\dfrac{BD}{CE} =$



圖一 Figure 1

2. 若 $Q > 0$ 並滿足 $\left|3Q - |1-2Q|\right| = 2$，求 Q 的值。

   If $Q > 0$ and satisfies $\left|3Q - |1-2Q|\right| = 2$, find the value of Q.

   $Q =$

3. 設 $xyzt = 1$。若 $R = \dfrac{1}{1+x+xy+xyz} + \dfrac{1}{1+y+yz+yzt} + \dfrac{1}{1+z+zt+ztx} + \dfrac{1}{1+t+tx+txy}$，求 R 的值。

   Let $xyzt = 1$. If $R = \dfrac{1}{1+x+xy+xyz} + \dfrac{1}{1+y+yz+yzt} + \dfrac{1}{1+z+zt+ztx} + \dfrac{1}{1+t+tx+txy}$, find the value of R.

   $R =$

4. 若 x_1、x_2、x_3、x_4 與 x_5 為正整數並滿足 $x_1 + x_2 + x_3 + x_4 + x_5 = x_1x_2x_3x_4x_5$，即是，五數之和等於五數之乘積，求 x_5 的最大值。

   If x_1, x_2, x_3, x_4 and x_5 are positive integers that satisfy $x_1 + x_2 + x_3 + x_4 + x_5 = x_1x_2x_3x_4x_5$, that is the sum is the product, find the maximum value of x_5.

   max x_5 =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2011 – 2012)
Final Event Spare (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設α及β為方程 $y^2 - 6y + 5 = 0$ 的實根。
設 m 為$|x - \alpha| + |x - \beta|$對任何實數 x 的最小值。求 m 的值。
Let α and β be the real roots of $y^2 - 6y + 5 = 0$. Let m be the minimum value of $|x - \alpha| + |x - \beta|$ over all real values of x. Find the value of m.

$m =$

2. 設α、β、γ 為實數且滿足 $\alpha + \beta + \gamma = 2$ 及 $\alpha\beta\gamma = 4$。
設 v 為 $|\alpha| + |\beta| + |\gamma|$ 的最小值，求 v 的值。
Let α, β, γ be real numbers satisfying $\alpha + \beta + \gamma = 2$ and $\alpha\beta\gamma = 4$.
Let v be the minimum value of $|\alpha| + |\beta| + |\gamma|$. Find the value of v .

$v =$

3. 設 $y = |x + 1| - 2|x| + |x - 2|$及$-1 \le x \le 2$。設$\alpha$為 y 的最大值，求 α 的值。
Let $y = |x + 1| - 2|x| + |x - 2|$ and $-1 \le x \le 2$. Let α be the maximum value of y .
Find the value of α .

$\alpha =$

4. 設 F 為方程 $x^2 + y^2 + z^2 + w^2 = 3(x + y + z + w)$的整數解的數目。求 F 的值。
Let F be the number of integral solutions of $x^2 + y^2 + z^2 + w^2 = 3(x + y + z + w)$.
Find the value of F .

$F =$

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

\+ Bonus score

Time Min. Sec.

Total score