Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event Sample (Individual)

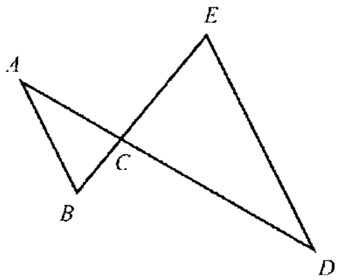
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 a、b、c 及 d 為方程 $x^4 - 15x^2 + 56 = 0$ 的根。
若 $P = a^2 + b^2 + c^2 + d^2$，求 P 的值。
Let a, b, c and d be the roots of the equation $x^4 - 15x^2 + 56 = 0$.
If $P = a^2 + b^2 + c^2 + d^2$, find the value of P .

$P =$

2. 如圖一，$AB = AC$ 及 $AB \parallel ED$。若 $\angle ABC = P°$ 及 $\angle ADE = Q°$，求 Q 的值。
In Figure 1, $AB = AC$ and $AB \parallel ED$. If $\angle ABC = P°$ and $\angle ADE = Q°$,
find the value of Q .

$Q =$



圖一
Figure 1

3. 設 $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^Q$ 及 $R = \sqrt{\dfrac{\log(1+F)}{\log 2}}$，求 R 的值。

Let $F = 1 + 2 + 2^2 + 2^3 + \ldots + 2^Q$ and $R = \sqrt{\dfrac{\log(1+F)}{\log 2}}$, find the value of R .

$R =$

4. 設 f(x)是一個函數使得對所有正整數 n，$f(n) = (n - 1)\,f(n - 1)$ 及 $f(1) \neq 0$。
若 $S = \dfrac{f(R)}{(R-1)f(R-3)}$，求 S 的值。
Let f(x) be a function such that $f(n) = (n - 1)\,f(n - 1)$ and $f(1) \neq 0$ for all positive integers n. If $S = \dfrac{f(R)}{(R-1)f(R-3)}$, find the value of S .

$S =$

<u>FOR OFFICIAL USE</u>

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
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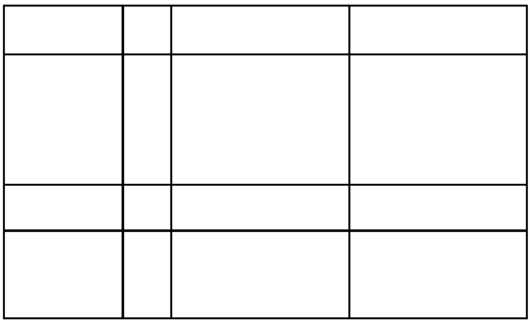
1. 圖一共有 a 個長方形，求 a 的值。
Figure 1 has a rectangles, find the value of a .

$a =$



圖一 Figure 1

2. 已知 111111 能被 7 整除。若 b 為 $\underbrace{111111\ldots111111}_{a個}$ 除以 7 的餘數，求 b 的值。
Given that 7 divides 111111.
If b is the remainder when $\underbrace{111111\ldots111111}_{a-times}$ is divided by 7, find the value of b .

$b =$

3. 若 c 為 $\left[(b-2)^{4b^2}+(b-1)^{2b^2}+b^{b^2}\right]$ 除以 3 的餘數，求 c 的數值。
If c is the remainder of $\left[(b-2)^{4b^2}+(b-1)^{2b^2}+b^{b^2}\right]$ divided by 3 , find the value of c .

$c =$

4. 若 $|x+1|+|y-1|+|z|=c$，求 $d=x^2+y^2+z^2$ 的值。
If $|x+1|+|y-1|+|z|=c$，find the value of $d=x^2+y^2+z^2$.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知函數 $f(x) = x^2 + rx + s$ 和 $g(x) = x^2 - 9x + 6$ 有以下特性：
$f(x)$ 的根之和是 $g(x)$ 的根之積，且 $f(x)$ 的根之積是 $g(x)$ 的根之和。
若 $f(x)$ 的最小值取值於 $x = a$，求 a 的值。
Given that functions $f(x) = x^2 + rx + s$ and $g(x) = x^2 - 9x + 6$ have the properties that the sum of roots of $f(x)$ is the product of the roots of $g(x)$, and the product of roots of $f(x)$ is the sum of roots of $g(x)$. If $f(x)$ attains its minimum at $x = a$, find the value of a.

$a =$

2. 一正方體的表面積是 b cm^2。
若它每一條邊的長度增加 3 cm，它的體積隨之增加 $(2b - a)$ cm^3，求 b 的值。
The surface area of a cube is b cm^2. If the length of each side is increased by 3 cm, its volume is increased by $(2b - a)$ cm^3, find the value of b.

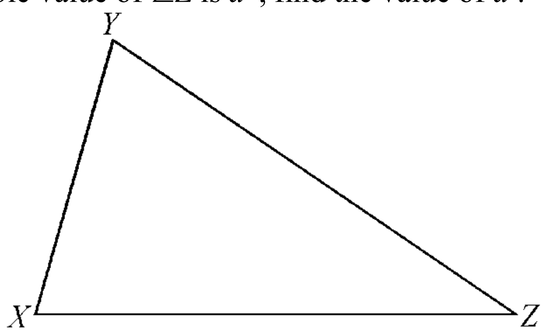
$b =$

3. 設 $f(1) = 3$，$f(2) = 5$ 且對所有正整數 n，$f(n + 2) = f(n + 1) + f(n)$。
當 $f(b)$ 除以 3 的餘數是 c，求 c 的值。
Let $f(1) = 3$, $f(2) = 5$ and $f(n + 2) = f(n + 1) + f(n)$ for positive integers n.
If c is the remainder of $f(b)$ divided by 3, find the value of c.

$c =$

4. 如圖二，三角形 XYZ 的角度滿足 $\angle Z \le \angle Y \le \angle X$ 且 $c\cdot\angle X = 6\cdot\angle Z$。
若$\angle Z$ 的最大可能值是 $d°$，求 d 的值。
In Figure 2, the angles of triangle XYZ satisfy $\angle Z \le \angle Y \le \angle X$ and $c\cdot\angle X = 6\cdot\angle Z$.
If the maximum possible value of $\angle Z$ is $d°$, find the value of d.

$d =$



圖二 Figure 2

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $a=\dfrac{\left(7+4\sqrt{3}\right)^{\frac{1}{2}}-\left(7-4\sqrt{3}\right)^{\frac{1}{2}}}{\sqrt{3}}$，求 a 的值。

   If $a=\dfrac{\left(7+4\sqrt{3}\right)^{\frac{1}{2}}-\left(7-4\sqrt{3}\right)^{\frac{1}{2}}}{\sqrt{3}}$, find the value of a.

   α =

2. 設 $\mathrm{f}(x)=x-a$ 及 $\mathrm{F}(x,y)=y^2+x$。如果 $b=\mathrm{F}(3,\mathrm{f}(4))$，求 b 的值。
   Suppose $\mathrm{f}(x)=x-a$ and $\mathrm{F}(x,y)=y^2+x$. If $b=\mathrm{F}(3,\mathrm{f}(4))$, find the value of b.

   b =

3. 已知 392 除以一個兩位正整數的餘數是 b，
   符合這個條件的兩位正整數共有 c 個，求 c 的值。
   The remainder when 392 is divided by a 2-digit positive integer is b.
   If c is the number of such 2-digit positive integers, find the value of c.

   c =

4. 若 x 為實數及 d 為函數 $y=\dfrac{3x^2+3x+c}{x^2+x+1}$ 的最大值，求 d 的值。

   If x is a real number and d is the maximum value of the function $y=\dfrac{3x^2+3x+c}{x^2+x+1}$,
   find the value of d.

   d =

FOR OFFICIAL USE

Score for accuracy × Mult. factor for speed =

Team No.

+ Bonus score

Time

Total score

Min. Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設實函數 f(x)對於所有實數 x 及 y 滿足 f(xy) = f(x)·f(y)，且 f(0) ≠ 0。
求 a = f(1)的值。
Let f(x) be a real value function that satisfies f(xy) = f(x)·f(y) for all real numbers x and y and f(0) ≠ 0. Find the value of a = f(1) .

a =

2. 設函數 $F(n)$滿足 $F(1) = F(2) = F(3) = a$ 及 $F(n+1) = \dfrac{F(n)\cdot F(n-1)+1}{F(n-2)}$，
其中 $n \geq 3$ 為正整數。求 $b = F(6)$的值。
Let $F(n)$ be a function with $F(1) = F(2) = F(3) = a$ and $F(n+1) = \dfrac{F(n)\cdot F(n-1)+1}{F(n-2)}$ for positive integer $n \geq 3$, find the value of $b = F(6)$.

b =

3. 若 $b-6$、$b-5$ 及 $b-4$ 為方程 $x^4 + rx^2 + sx + t = 0$ 的根，求 $c = r + t$ 的值。
If $b-6$, $b-5$, $b-4$ are three roots of the equation $x^4 + rx^2 + sx + t = 0$,
find the value of $c = r + t$.

c =

4. 設(x_0, y_0) 是以下方程組的一個解：
$$\begin{cases} xy = 6 \\ x^2y + xy^2 + x + y + c = 2 \end{cases}$$
求 $d = x_0^2 + y_0^2$ 的值。
Suppose that (x_0, y_0) is a solution of the system:
$$\begin{cases} xy = 6 \\ x^2y + xy^2 + x + y + c = 2 \end{cases}$$
Find the value of $d = x_0^2 + y_0^2$.

d =

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event Sample (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知三角形三邊的長度分別是 a cm、2 cm 及 b cm，其中 a 和 b 是整數且 $a \le 2 \le b$。若有 q 種不全等的三角形滿足上述條件，求 q 的值。
Given some triangles with side lengths a cm, 2 cm and b cm, where a and b are integers and $a \le 2 \le b$. If there are q non-congruent classes of triangles satisfying the above conditions, find the value of q.

$q =$

2. 已知方程 $|x| - \frac{4}{x} = \frac{3|x|}{x}$ 有 k 個相異實根，求 k 的值。
Given that the equation $|x| - \frac{4}{x} = \frac{3|x|}{x}$ has k distinct real root(s), find the value of k.

$k =$

3. 已知 x 及 y 為非零實數且滿足方程 $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ 及 $x - y = 7$。
若 $w = x + y$，求 w 的值。
Given that x and y are non-zero real numbers satisfying the equations $\frac{\sqrt{x}}{\sqrt{y}} - \frac{\sqrt{y}}{\sqrt{x}} = \frac{7}{12}$ and $x - y = 7$. If $w = x + y$, find the value of w.

$w =$

4. 已知 x 及 y 為實數且 $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$。設 $p = |x| + |y|$，求 p 的值。
Given that x and y are real numbers and $\left|x - \frac{1}{2}\right| + \sqrt{y^2 - 1} = 0$.
Let $p = |x| + |y|$, find the value of p.

$p =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 求 $(2^{13}+1)(2^{14}+1)(2^{15}+1)(2^{16}+1)$ 的個位數字。
Find the units digit of $(2^{13}+1)(2^{14}+1)(2^{15}+1)(2^{16}+1)$.

unit digit =

2. 求 $16 \div (0.40 + 0.41 + 0.42 + \dots + 0.59)$ 的值的整數部分。
Find the integral part of $16 \div (0.40 + 0.41 + 0.42 + \dots + 0.59)$.

integral part

3. 從 1、2、4、6、7 中選三個數字組成三位數。
這些三位數有多少個能被 3 整除？
Choose three digits from 1, 2, 4, 6, 7 to construct three-digit numbers.
Of these three-digit numbers, how many of them are divisible by 3?

4. 用 1、2、3、4、5、6 組成一個位數：$ABCDEF$，使得 A 能被 1 整除，AB 能被 2 整除，ABC 能被 3 整除，$ABCD$ 能被 4 整除，$ABCDE$ 能被 5 整除，及 $ABCDEF$ 能被 6 整除。求 A 的最大值。
Using numbers: 1, 2, 3, 4, 5, 6 to form a six-digit number: $ABCDEF$ such that A is divisible by 1, AB is divisible by 2, ABC is divisible by 3, $ABCD$ is divisible by 4, $ABCDE$ is divisible by 5, $ABCDEF$ is divisible by 6. Find the greatest value of A.

Greatest A

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

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除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $4^3 + 4^r + 4^4$ 是一平方數，其中 r 是正整數，求 r 的最小值。
If $4^3 + 4^r + 4^4$ is a perfect square and r is a positive integer, find the minimum value of r.

minimum r

2. 三男 B_1、B_2、B_3 和三女 G_1、G_2、G_3 就坐一排座位，並滿足以下兩個條件：
1) 一男不會坐在另一男旁邊及一女不會坐在另一女旁邊
2) B_1 必須坐在 G_1 旁邊
若 s 是這樣就坐的排列數量，求 s 的值。
Three boys B_1, B_2, B_3 and three girls G_1, G_2, G_3 are to be seated in a row according to the following rules:
1) A boy will not sit next to another boy and a girl will not sit next to another girl ,
2) Boy B_1 must sit next to girl G_1
If s is the number of different such seating arrangements, find the value of s.

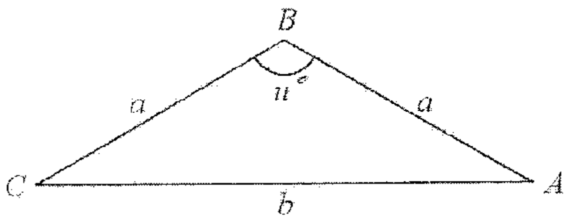
$s =$

3. 設 $f(x) = \dfrac{x+a}{x^2 + \frac{1}{2}}$，$x$ 為實數且 $f(x)$ 的最大值和最小值分別是 $\frac{1}{2}$ 和 -1。
若 $t = f(0)$，求 t 的值。
Let $f(x) = \dfrac{x+a}{x^2 + \frac{1}{2}}$, where x is a real number and the maximum value of $f(x)$ is $\frac{1}{2}$ and the minimum value of $f(x)$ is -1. If $t = f(0)$, find the value of t.

$t =$

4. 在圖三，ABC 是一等腰三角形，其中 $\angle ABC = u°$，$AB = BC = a$ 和 $AC = b$。
若二次方程 $ax^2 - \sqrt{2} \cdot bx + a = 0$ 有兩個實根，它們的絕對差為 $\sqrt{2}$，求 u 的值。
In Figure 3, ABC is an isosceles triangle with $\angle ABC = u°$, $AB = BC = a$ and $AC = b$.
If the quadratic equation $ax^2 - \sqrt{2} \cdot bx + a = 0$ has two real roots, whose absolute difference is $\sqrt{2}$, find the value of u.

$u =$



圖三 Figure 3

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 m 和 n 是正整數且 $m^2 - n^2 = 43$，求 $m^3 - n^3$ 的值。
If m and n are positive integers with $m^2 - n^2 = 43$, find the value of $m^3 - n^3$.

$m^3 - n^3 =$

2. 設 x_1、x_2、…、x_{10} 為非零整數，且滿足 $-1 \le x_i \le 2$，其中 $i = 1$、2、…、10。
若 $x_1 + x_2 + \ldots + x_{10} = 11$，求 $x_1^2 + x_2^2 + \cdots + x_{10}^2$ 的最大可能值。
Let $x_1, x_2, \ldots, x_{10}$ be non-zero integers satisfying $-1 \le x_i \le 2$ for $i = 1, 2, \ldots, 10$.
If $x_1 + x_2 + \cdots + x_{10} = 11$, find the maximum possible value for $x_1^2 + x_2^2 + \cdots + x_{10}^2$.

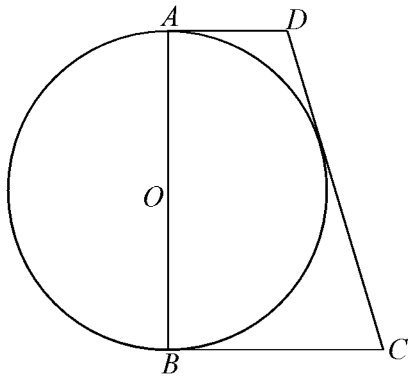
Maximum

3. 若 $f(n) = a^n + b^n$，其中 n 是正整數且 $f(3) = [f(1)]^3 + f(1)$，求 $a \cdot b$ 的值。
If $f(n) = a^n + b^n$, where n is a positive integer and $f(3) = [f(1)]^3 + f(1)$, find the value of $a \cdot b$.

$a \cdot b =$

4. 在圖四，AD、BC 和 CD 是以 O 作圓心且直徑 $AB = 12$ 的圓的切綫。
若 $AD = 4$，求 BC 的值。
In Figure 4, AD, BC and CD are tangents to the circle with centre at O and diameter $AB = 12$. If $AD = 4$, find the value of BC.

$BC =$



圖四 Figure 4

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (2012 – 2013)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 P 為整數 3,659,893,456,789,325,678 與 342,973,489,379,256 的乘積，求 P 的位數。
In P be the product of 3,659,893,456,789,325,678 and 342,973,489,379,256 , find the number of digits of P .

no. of digits

2. 若 $\frac{1}{4}+4\left(\frac{1}{2013}+\frac{1}{x}\right)=\frac{7}{4}$，求 $1872+48\times\left(\frac{2013x}{x+2013}\right)$ 的值。
If $\frac{1}{4}+4\left(\frac{1}{2013}+\frac{1}{x}\right)=\frac{7}{4}$, find the value of $1872+48\times\left(\frac{2013x}{x+2013}\right)$.

3. 有一個整數被 10 除，餘數為 9；被 9 除，餘數為 8；被 8 除，餘數為 7；等等直至被 2 除，餘數為 1。求此整數的最小值。
The remainders of an integer when divided by 10, 9, 8, $\cdots$, 2 are 9, 8, 7, $\cdots$, 1 respectively. Find the smallest such an integer .

4. 如圖五，A、B、C、D、E 代表不同的個位數字。求 $A+B+C+D+E$ 的值。
In Figure 5, A, B, C, D, E represent different digits.
Find the value of $A+B+C+D+E$.

$$\begin{array}{r} ABCDE \\ \times \qquad\quad 9 \\ \hline 1AAA0E \end{array}$$

圖五 Figure 5

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+ Bonus score				Time		
			Total score					Min.	Sec.