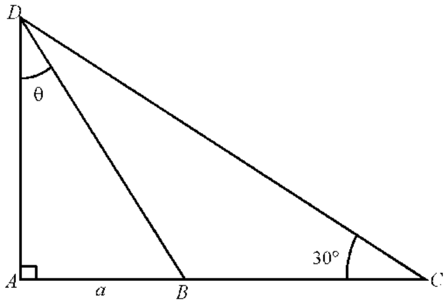
Hong Kong Mathematics Olympiad 2012-2013
Heat Event (Individual)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 化簡 $\sqrt{94-2\sqrt{2013}}$。
   Simplify $\sqrt{94-2\sqrt{2013}}$.

2. 一個平行四邊形可被分成 178 個邊長為 1 單位的等邊三角形，若該平行四邊形的周界為 P 單位，求 P 的最大值。
   A parallelogram is cut into 178 pieces of equilateral triangles with sides 1 unit. If the perimeter of the parallelogram is P units, find the maximum value of P .

3. 圖一所示為一直角三角形 ACD，其中 B 是 AC 上的點且 $BC = 2AB$。已知 $AB = a$ 及 $\angle ACD = 30°$，求 θ 的值。
   Figure 1 shows a right-angled triangle ACD where B is a point on AC and $BC = 2AB$.
   Given that $AB = a$ and $\angle ACD = 30°$, find the value of θ .



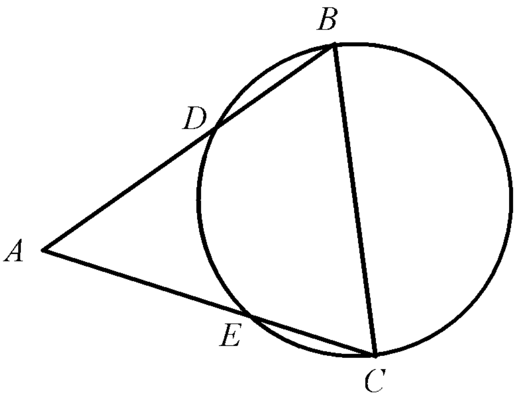
圖一 Figure 1

4. 已知 $x^2 + 399 = 2^y$，其中 x、y 為正整數。求 x 的值。
   Given that $x^2 + 399 = 2^y$, where x, y are positive integers. Find the value of x .

5. 已知 $y = (x + 1)(x + 2)(x + 3)(x + 4) + 2013$，求 y 的最小值。
   Given that $y = (x + 1)(x + 2)(x + 3)(x + 4) + 2013$, find the minimum value of y.

6. 從一個有 n 條邊的凸多邊形中，選取其中一隻內角。若餘下的 $n - 1$ 隻內角之和是 2013°，求 n 的值。
   In a convex polygon with n sides, one interior angle is selected. If the sum of the remaining n – 1 interior angle is 2013°, find the value of n .

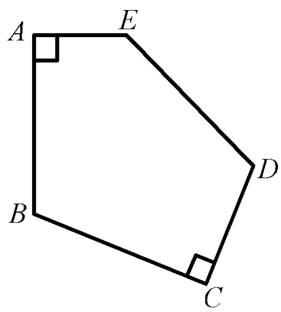
7. 圖二所示為一通過 B 點及 C 點的圓，而 A 點則在圓之外。已知 BC 是圓的直徑，AB 及 AC 分別與圓相交於 D 點及 E 點，且 $\angle BAC = 45°$，求 $\frac{\Delta ADE \text{ 的面積}}{BCED \text{ 的面積}}$。
   Figure 2 shows a circle passes through two points B and C, and a point A is lying outside the circle. Given that BC is a diameter of the circle, AB and AC intersect the circle at D and E respectively and $\angle BAC = 45°$, find $\frac{\text{area of } \Delta ADE}{\text{area of } BCED}$.



圖二 Figure 2

8. 解 $\sqrt{31-\sqrt{31+x}} = x$。
   Solve $\sqrt{31-\sqrt{31+x}} = x$.

9. 圖三所示為五邊形 $ABCDE$。$AB = BC = DE = AE + CD = 3$，且 $\angle A = \angle C = 90°$，求該五邊形的面積。
   Figure 3 shows a pentagon $ABCDE$. $AB = BC = DE = AE + CD = 3$ and $\angle A = \angle C = 90°$, find the area of the pentagon.



圖三 Figure 3

10. 若 a 及 b 為實數，且 $a^2 + b^2 = a + b$。求 $a + b$ 的最大值。
    If a and b are real numbers, and $a^2 + b^2 = a + b$. Find the maximum value of $a + b$.

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2012-2013
Heat Event (Group)

除非特別聲明，答案須用數字表達，並化至最簡。 時限：20 分鐘
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每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 已知一個直角三角形三邊的長度皆為整數，且其中兩邊的長度為方程 $x^2 - (m + 2)x + 4m = 0$ 的根。求第三邊長度的最大值。

   Given that the length of the sides of a right-angled triangle are integers, and two of them are the roots of the equation $x^2 - (m + 2)x + 4m = 0$.

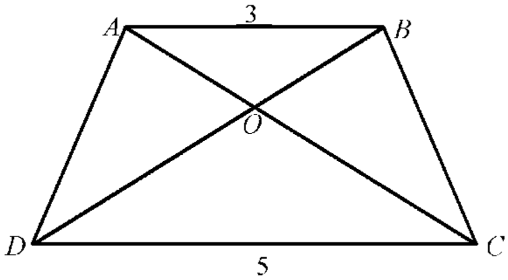
   Find the maximum length of the third side of the triangle.

2. 圖一所示為一梯形 $ABCD$，其中 $AB = 3$、$CD = 5$ 及 AC、BD 相交於點 O。若 ΔAOB 的面積是 27，求梯形 $ABCD$ 的面積。

   Figure 1 shows a trapezium $ABCD$, where $AB = 3$, $CD = 5$ and the diagonals AC and BD meet at O.

   If the area of ΔAOB is 27,

   find the area of the trapezium $ABCD$.



圖一 Figure 1

3. 設 x 及 y 為實數使得 $x^2 + xy + y^2 = 2013$。求 $x^2 - xy + y^2$ 的最大值。

   Let x and y be real numbers such that $x^2 + xy + y^2 = 2013$.

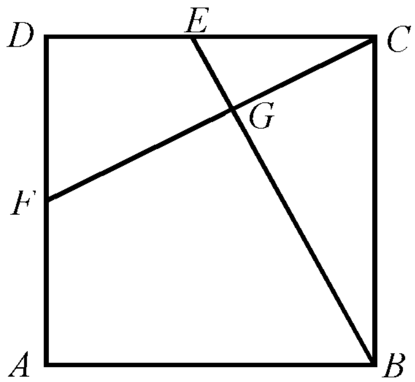
   Find the maximum value of $x^2 - xy + y^2$.

4. 若 α、β 是方程 $x^2 + 2013x + 5 = 0$ 的根，求 $(\alpha^2 + 2011\alpha + 3)(\beta^2 + 2015\beta + 7)$ 的值。

   If α, β are roots of $x^2 + 2013x + 5 = 0$, find the value of $(\alpha^2 + 2011\alpha + 3)(\beta^2 + 2015\beta + 7)$.

5. 如圖二所示，$ABCD$ 為一個邊長為 10 單位的正方形，E 及 F 分別為 CD 及 AD 的中點，BE 及 FC 相交於 G。求 AG 的長度。

   As shown in Figure 2, $ABCD$ is a square of side 10 units, E and F are the mid-points of CD and AD respectively, BE and FC intersect at G. Find the length of AG.



圖二 Figure 2

6. 若 a 及 b 為正實數，且方程 $x^2 + ax + 2b = 0$ 及 $x^2 + 2bx + a = 0$ 都有實數根。求 $a + b$ 的最小值。

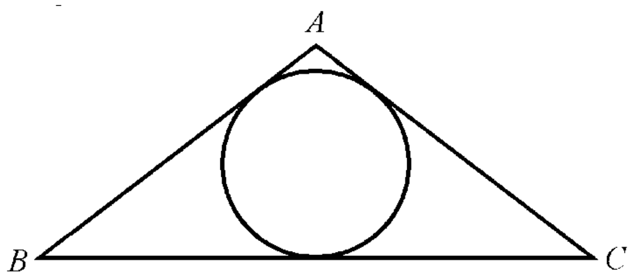
Let a and b are positive real numbers, and the equations $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ have real roots. Find the minimum value of $a + b$.

7. 已知 ΔABC 的三邊的長度組成一個等差數列，且為方程 $x^3 - 12x^2 + 47x - 60 = 0$ 的根，求 ΔABC 的面積。

Given that the length of the three sides of ΔABC form an arithmetic sequence, and are the roots of the equation $x^3 - 12x^2 + 47x - 60 = 0$, find the area of ΔABC.

8. 圖三中，ΔABC 為一等腰三角形，其中 $AB = AC$，$BC = 240$。已知 ΔABC 的內接圓的半徑是 24，求 AB 的長度。

In Figure 3, ΔABC is an isosceles triangle with $AB = AC$, $BC = 240$. The radius of the inscribed circle of ΔABC is 24.
Find the length of AB.



圖三 Figure 3

9. 從 1、2、3、⋯、2012、2013 中最多可取出多少個數，使得在取出的數中任意兩數之和都不是這兩個數之差的倍數？

At most how many numbers can be taken from the set of integers: 1, 2, 3, ⋯ , 2012, 2013 such that the sum of any two numbers taken out from the set is not a multiple of the difference between the two numbers?

10. 對所有正整數 n，定義函數 f 為

(i) $f(1) = 2012$,

(ii) $f(1) + f(2) + \cdots + f(n-1) + f(n) = n^2 f(n)$，$n > 1$

求 f(2012) 的值。

For all positive integers n, define a function f as

(i) $f(1) = 2012$,

(ii) $f(1) + f(2) + \cdots + f(n-1) + f(n) = n^2 f(n)$，$n > 1$.

Find the value of f(2012) .

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2012 – 2013
Heat Event (Geometric Construction)
香港數學競賽 2012 – 2013
初賽(幾何作圖)

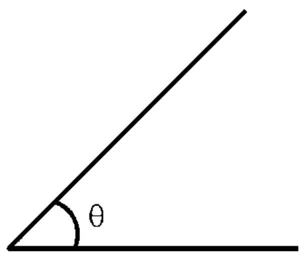
每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

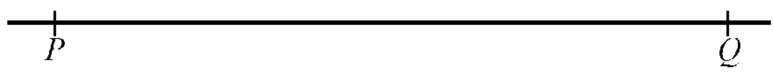
School Code: ______________________________

School Name: ______________________________

第一題 Question No. 1

下圖所示為綫段 PQ 及角 θ。試構作一個等腰三角形 PQR，其中 $PQ = PR$ 及 $\angle QPR = \theta$。
Line segment PQ and an angle of size θ are given below. Construct the isosceles triangle PQR with $PQ = PR$ and $\angle QPR = \theta$.





Hong Kong Mathematics Olympiad 2012 – 2013
Heat Event (Geometric Construction)
香港數學競賽 2012 – 2013
初賽(幾何作圖)

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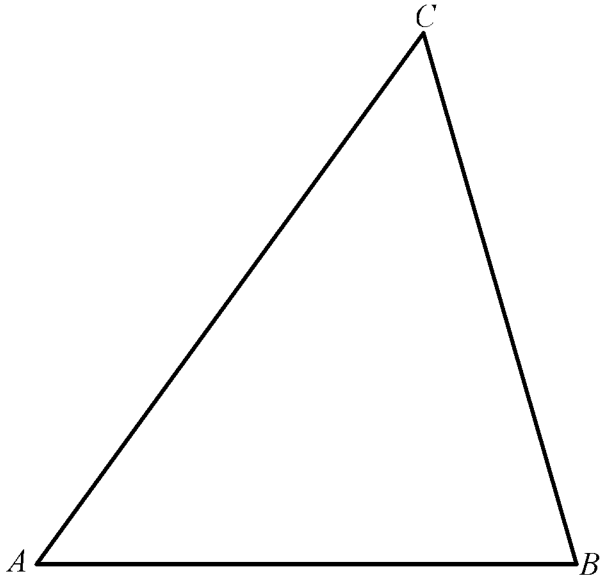
第二題 Question No. 2

試構作一個面積與下圖所示的 ΔABC 面積相等的長方形，長方形其中一邊為 AB。

Construct a rectangle with AB as one of its sides and with area equal to that of ΔABC below.



Hong Kong Mathematics Olympiad 2012 – 2013
Heat Event (Geometric Construction)
香港數學競賽 2012 – 2013
初賽(幾何作圖)

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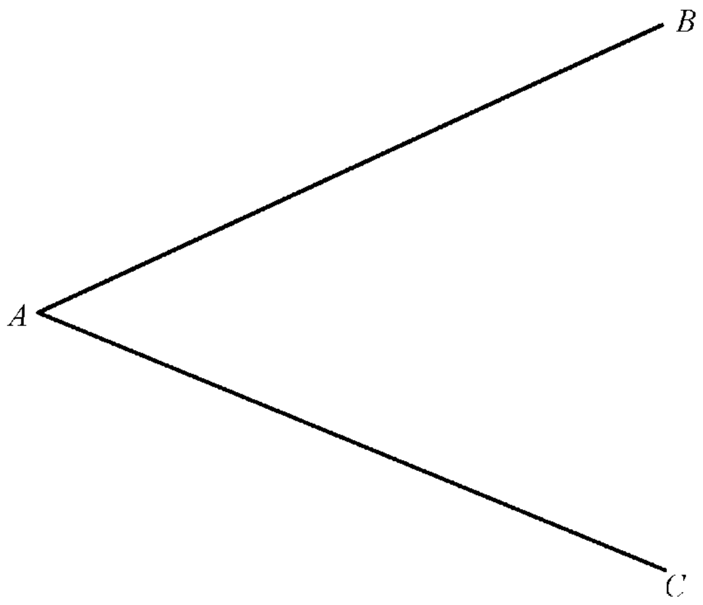
School Code: ______________________________

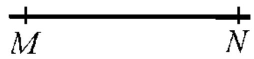
School Name: ______________________________

第三題 Question No. 3

下圖所示為兩相交於 A 點的綫段 AB 及 AC。試構作一半徑等於綫段 MN 的圓使得 AB 及 AC 均為該圓的切綫。

The figure below shows two straight lines AB and AC intersecting at the point A. Construct a circle with radius equal to the line segment MN so that AB and AC are tangents to the circle.





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