Hong Kong Mathematics Olympiad 2013-2014
Heat Event (Individual)

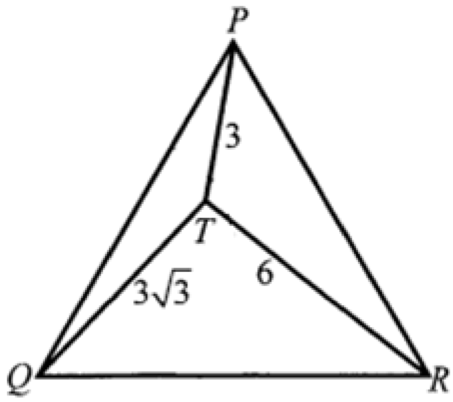
除非特別聲明，答案須用數字表達，並化至最簡。 時限：40 分鐘
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 40 minutes

1. 已知 a、b、$c > 0$ 且 $\begin{cases} \dfrac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}} = 2 \\ \dfrac{\sqrt{bc}}{\sqrt{b}+\sqrt{c}} = 3 \\ \dfrac{\sqrt{ca}}{\sqrt{c}+\sqrt{a}} = 5 \end{cases}$，求 $\dfrac{a}{\sqrt{bc}}$ 的值。

   Given that $a, b, c > 0$ and $\begin{cases} \dfrac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}} = 2 \\ \dfrac{\sqrt{bc}}{\sqrt{b}+\sqrt{c}} = 3 \\ \dfrac{\sqrt{ca}}{\sqrt{c}+\sqrt{a}} = 5 \end{cases}$. Find the value of $\dfrac{a}{\sqrt{bc}}$.

2. 已知 $a = 2014x + 2011$，$b = 2014x + 2013$ 及 $c = 2014x + 2015$。
   求 $a^2 + b^2 + c^2 - ab - bc - ca$ 的值。
   Given that $a = 2014x + 2011$, $b = 2014x + 2013$ and $c = 2014x + 2015$.
   Find the value of $a^2 + b^2 + c^2 - ab - bc - ca$.

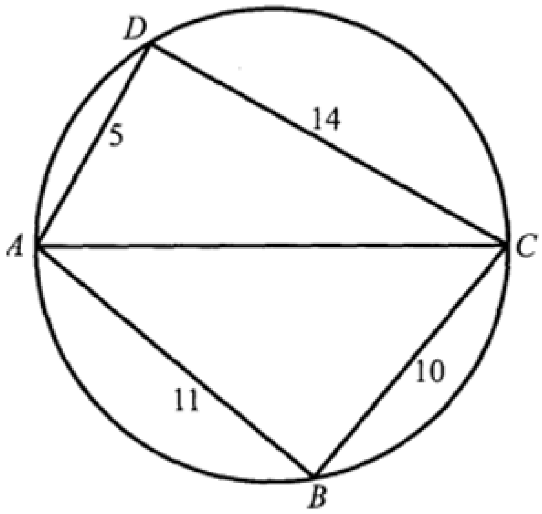
3. 如圖一所示，T 為等邊三角形 PQR 內一點，
   其中 $TP = 3$、$TQ = 3\sqrt{3}$ 及 $TR = 6$。求 $\angle PTR$ 的值。
   As shown in Figure 1, a point T lies in an equilateral triangle PQR such that $TP = 3$, $TQ = 3\sqrt{3}$ and $TR = 6$.
   Find the value of $\angle PTR$.



圖一 Figure 1

4. 設 α 及 β 為二次方程 $x^2 - 14x + 1 = 0$ 的根。求 $\dfrac{\alpha^2}{\beta^2+1} + \dfrac{\beta^2}{\alpha^2+1}$ 的值。
   Let α and β be the roots of the quadratic equation $x^2 - 14x + 1 = 0$.
   Find the value of $\dfrac{\alpha^2}{\beta^2+1} + \dfrac{\beta^2}{\alpha^2+1}$.

5. 如圖二所示，$ABCD$ 為圓內接四邊形，其中 $AD = 5$、$DC = 14$、$BC = 10$ 及 $AB = 11$。求四邊形 $ABCD$ 的面積。
As shown in Figure 2, $ABCD$ is a cyclic quadrilateral, where $AD = 5$, $DC = 14$, $BC = 10$ and $AB = 11$. Find the area of quadrilateral $ABCD$.



圖二 Figure 2

6. 設 n 為正整數，且 $n < 1000$。若 $(n-1)^2$ 整除 $(n^{2014}-1)$，求 n 的最大值。
Let n be a positive integer and $n < 1000$.
If $(n^{2014}-1)$ is divisible by $(n-1)^2$, find the maximum value of n.

7. 若 $x^3 + x^2 + x + 1 = 0$，
求 $x^{-2014} + x^{-2013} + x^{-2012} + \cdots + x^{-1} + 1 + x + x^2 + \cdots + x^{2013} + x^{2014}$ 的值。
If $x^3 + x^2 + x + 1 = 0$,
find the value of $x^{-2014} + x^{-2013} + x^{-2012} + \cdots + x^{-1} + 1 + x + x^2 + \cdots + x^{2013} + x^{2014}$.

8. 設 $\overline{xy} = 10x + y$。若 $\overline{xy} + \overline{yx}$ 為一個平方數，這樣的數有多少個？
Let $\overline{xy} = 10x + y$. If $\overline{xy} + \overline{yx}$ is a square number, how many numbers of this kind exist?

9. 已知 x、y 及 z 為正實數，且 $xyz = 64$。
設 $S = x + y + z$，求當 $4x^2 + 2xy + y^2 + 6z$ 的值為最小時，S 的值。
Given that x, y and z are positive real numbers such that $xyz = 64$.
If $S = x + y + z$, find the value of S when $4x^2 + 2xy + y^2 + 6z$ is a minimum.

10. 已知 ΔABC 為一銳角三角形，其中 $\angle A > \angle B > \angle C$。
若 $x°$ 為 $\angle A - \angle B$、$\angle B - \angle C$ 及 $90° - \angle A$ 中的最小值，求 x 的最大值。
Given that ΔABC is an acute triangle, where $\angle A > \angle B > \angle C$.
If $x°$ is the minimum of $\angle A - \angle B$, $\angle B - \angle C$ and $90° - \angle A$, find the maximum value of x.

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2013-2014
Heat Event (Group)

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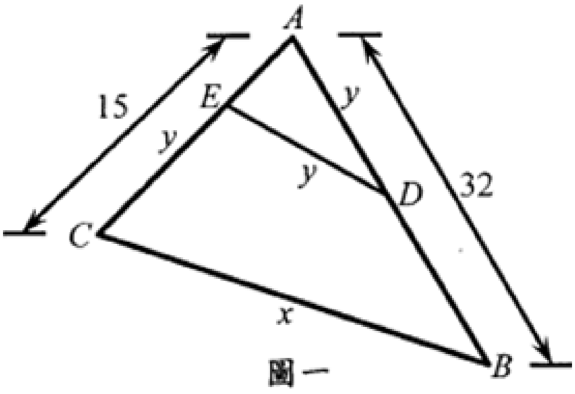
每題正確答案得一分。Each correct answer will be awarded 1 mark. Time allowed: 20 minutes

1. 已知 $\sqrt{2014-x^2}-\sqrt{2004-x^2}=2$。求 $\sqrt{2014-x^2}+\sqrt{2004-x^2}$ 的值。

Given that $\sqrt{2014-x^2}-\sqrt{2004-x^2}=2$, find the value of $\sqrt{2014-x^2}+\sqrt{2004-x^2}$.

2. 圖一顯示 ΔABC 中，$AB = 32$、$AC = 15$ 及 $BC = x$，其中 x 為一個正整數。假設 AB 及 AC 分別有一點 D 及 E 使得 $AD = DE = EC = y$，其中 y 為一個正整數。求 x 的值。

Figure 1 shows a ΔABC, $AB = 32$, $AC = 15$ and $BC = x$, where x is a positive integer. If there are points D and E lying on AB and AC respectively such that $AD = DE = EC = y$, where y is a positive integer. Find the value of x.



圖一

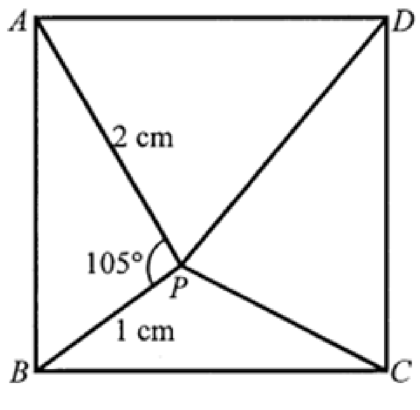
Figure 1

3. 若 $0° \le \theta \le 180°$ 及 $\cos\theta + \sin\theta = \frac{7}{13}$，求 $\cos\theta + \cos^3\theta + \cos^5\theta + \cdots$ 的值。

If $0° \le \theta \le 180°$ and $\cos\theta + \sin\theta = \frac{7}{13}$, find the value of $\cos\theta + \cos^3\theta + \cos^5\theta + \cdots$.

4. 如圖二所示，$ABCD$ 為一正方形。P 為 $ABCD$ 內的一點使得 $AP = 2$ cm、$BP = 1$ cm 及 $\angle APB = 105°$。若 $CP^2 + DP^2 = x$ cm^2，求 x 的值。

As shown in Figure 2, $ABCD$ is a square. P is a point lies in $ABCD$ such that $AP = 2$ cm, $BP = 1$ cm and $\angle APB = 105°$. If $CP^2 + DP^2 = x$ cm^2, find the value of x.

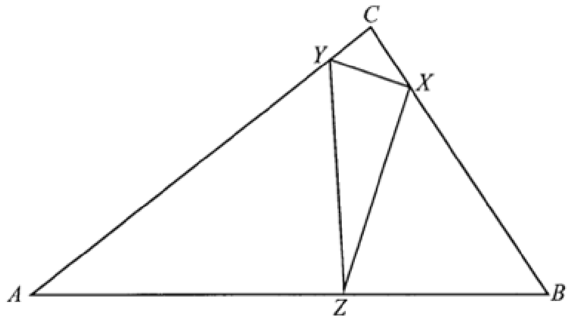


圖二 Figure 2

5. 若 x、y 是實數，且 $x^2 + 3y^2 = 6x + 7$，求 $x^2 + y^2$ 的極大值。

If x, y are real numbers and $x^2 + 3y^2 = 6x + 7$, find the maximum value of $x^2 + y^2$.

6. 如圖三所示，在ΔABC 中，X、Y 及 Z 為分別位於 BC、CA 及 AB 的點使得$\angle AZY = \angle BZX$、$\angle BXZ = \angle CXY$ 及$\angle CYX = \angle AYZ$。若 $AB = 10$、$BC = 6$ 及 $CA = 9$，求 AZ 的長度。
As shown in Figure 3, X, Y and Z are points on BC, CA and AB of ΔABC respectively such that $\angle AZY = \angle BZX$, $\angle BXZ = \angle CXY$ and $\angle CYX = \angle AYZ$. If $AB = 10$, $BC = 6$ and $CA = 9$, find the length of AZ.



圖三 Figure 3

7. 已知 a、b、c 及 d 為四個不相同的數，且 $(a+c)(a+d)=1$ 及 $(b+c)(b+d)=1$，求 $(a+c)(b+c)$ 的值。
Given that a, b, c and d are four distinct numbers, where $(a+c)(a+d)=1$ and $(b+c)(b+d)=1$. Find the value of $(a+c)(b+c)$.

8. 設 $a_1 = 215$，$a_2 = 2014$ 及 $a_{n+2} = 3a_{n+1} - 2a_n$，其中 n 為一正整數。求 $a_{2014} - 2a_{2013}$ 的值。
Let $a_1 = 215$, $a_2 = 2014$ and $a_{n+2} = 3a_{n+1} - 2a_n$, where n is a positive integer. Find the value of $a_{2014} - 2a_{2013}$.

9. 已知函數 $y = \sin^2 x - 4\sin x + m$ 的極小值為 $\frac{-8}{3}$，求 m^y 的極小值。
Given that the minimum value of the function $y = \sin^2 x - 4\sin x + m$ is $\frac{-8}{3}$.
Find the minimum value of m^y.

10. 已知 $\tan\left(\frac{90^\circ}{\tan x}\right)\times\tan\left(90^\circ \tan x\right)=1$ 及 $1 < \tan x < 3$。求 $\tan x$ 的值。
Given that $\tan\left(\frac{90^\circ}{\tan x}\right)\times\tan\left(90^\circ \tan x\right)=1$ and $1 < \tan x < 3$. Find the value of $\tan x$.

***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad 2013 – 2014
Heat Event (Geometric Construction)
香港數學競賽 2013 – 2014
初賽(幾何作圖)

每隊必須列出詳細所有步驟(包括作圖步驟)。 時限：20 分鐘
All working (including geometric drawing) must be clearly shown.
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

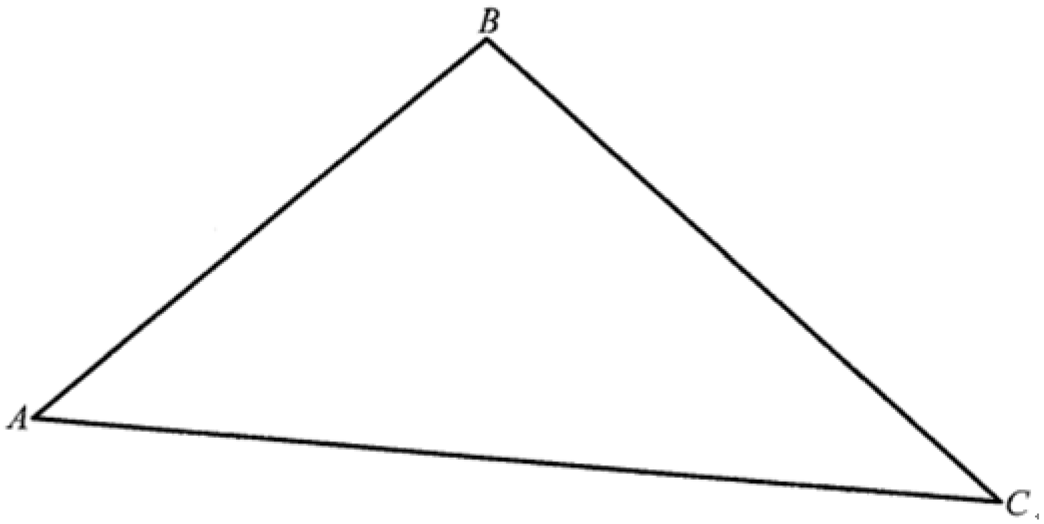
School Code: ____
School Name: ____

第一題 Question No. 1

圖一所示為一個ΔABC。

試在該三角形內，構作一個圓心為 O 的圓，使三角形三條邊均為該圓的切綫。

Figure 1 shows a ΔABC. Construct a circle with centre O inside the triangle such that the three sides of the triangle are tangents to the circle.



圖一 Figure 1

Hong Kong Mathematics Olympiad 2013 – 2014
Heat Event (Geometric Construction)
香港數學競賽 2013 – 2014
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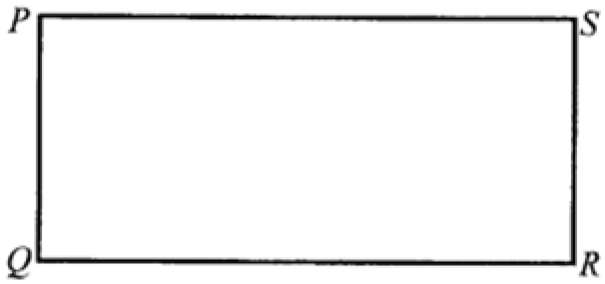
School Code: ____________________

School Name: ____________________

第二題 Question No. 2

圖二所示為一個長方形 *PQRS*。試構作一個面積與該長方形面積相等的正方形。

Figure 2 shows a rectangle *PQRS*. Construct a square of area equal to that of a rectangle.



圖二 Figure 2

Hong Kong Mathematics Olympiad 2013 – 2014
Heat Event (Geometric Construction)
香港數學競賽 2013 – 2014
初賽(幾何作圖)

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All working (including geometric drawing) must be clearly shown.
此部份滿分為十分。The full marks of this part is 10 marks. Time allowed: 20 minutes

School Code: ____________________

School Name: ____________________

第三題 Question No. 3

圖三所示為兩綫段 AB 及 AC 相交於 A 點。試在它們之間構作兩個大小不同的圓使得

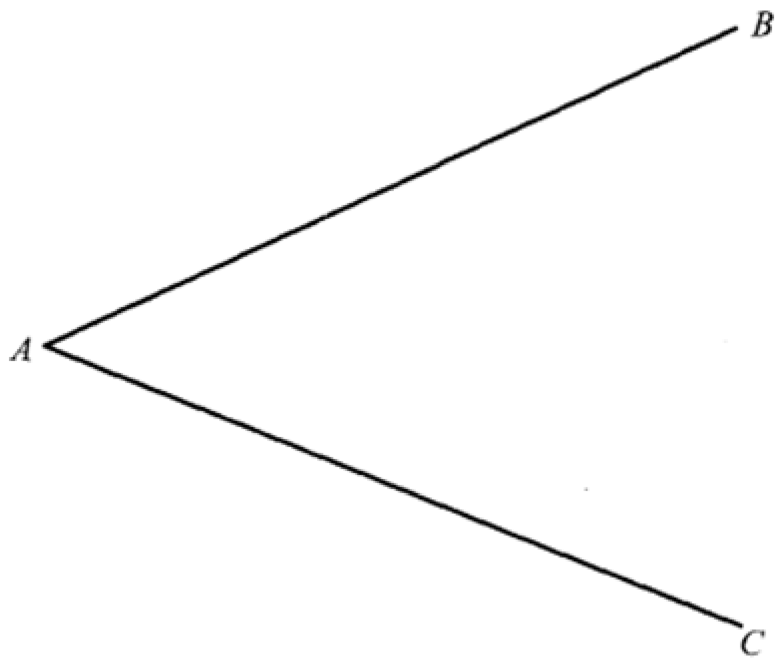
(i) 該兩圓相切於一點；及

(ii) 綫段 AB 及 AC 均為該兩圓的切綫。

Figure 3 shows two line segments AB and AC intersecting at the point A . Construct two circles of different sizes between them such that

(i) They touch each other at a point; and

(ii) the line segments AB and AC are tangents to both circles.



圖三 Figure 3

***** 試卷完 End of Paper *****