

第三十六屆香港數學競賽 (2018/19)

決賽規則

1. 競賽共分八項，個人及團體各佔四項。
2. 每隊由已報名參加初賽的同學組成。其中任何四位可參加「個人項目」；又其中任何四位可參加「團體項目」。不足四位同學的隊伍將不獲准出賽。
3. 每隊隊員必須穿著整齊校服，並由負責老師帶領，於上午9時正或以前向會場接待處註冊，同時必須出示身分證/學生證明文件，否則將被撤銷參賽資格。
4. 粵語將會被採用為指示語言。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英並列。
5. 每一「個人項目」包括四部份。每一隊員回答其中一部份，其他隊友不得從旁協助，否則此項目所得分數會被取消。
6. 「個人項目」中，四部份互有關連。解答第二部份之隊員需利用第一部份之答案，如此類推。
7. 每一「團體項目」亦包括四部份。但各部份不一定相關，且可由全隊共同作答。各隊員可進行討論，但必須將聲浪降至最低。
8. 比賽時，參賽者不可使用計算工具，違例者將被取消資格或扣分。
9. 參賽者如有攜帶電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，應把它關掉，並放入手提包內或座位的椅下，否則大會有權取消該隊參賽資格。
10. 除另有聲明外，所有答案須為數字，並應化簡，但無需呈交證明及算草。
11. 每一項目限時五分鐘。
12. 計分辦法如下：

| (甲) 準確分: | 個人項目 | 積分 | 團體項目 | 積分 |
|----------|--------|----|---------|----|
| | 答對第一部分 | 1 | 答對任何一部分 | 2 |
| | 答對第二部分 | 2 | 答對任何兩部分 | 4 |
| | 答對第三部分 | 3 | 答對任何三部分 | 7 |
| | 答對第四部分 | 4 | 答對所有四部分 | 10 |
| | 合共 | 10 | | |

| (乙) 快捷分 | 積分所乘倍數 |
|--|----------|
| 參賽隊伍完成並交出答案的時間 < 1 分鐘 | 4 |
| 1 分鐘 ≤ 參賽隊伍完成並交出答案的時間 < 2 分鐘 | 3 |
| 2 分鐘 ≤ 參賽隊伍完成並交出答案的時間 < 3 分鐘 | 2 |
| 參賽隊伍完成並交出答案的時間 ≥ 3 分鐘 | 1 |

(丙) 獎勵分

任何一隊在某一個人/團體項目競賽中，若全部答對時，可額外獲得 20 分。

(丁) 每項目之總分

準確分×倍數 + 獎勵分

13. 如有任何疑問，參賽者須於最後一項個人/團體賽完畢後 10 分鐘內向評判團提出。所有提出之疑問，將由評判團作最後裁決。
14. 得分最高之三隊將獲得獎盃及獎品。冠軍學校可保存總冠軍盾牌至下一屆香港數學競賽。
15. 總成績將由評判團作最後裁決。

The Thirty-sixth Hong Kong Mathematics Olympiad (2018/19)

Regulations (Final Events)

1. The competition consists of 8 events, which are divided into 4 individual events and 4 group events.
2. Each participating team should consist of students who have enrolled in the heats. Any 4 of them may take part in the individual event and any 4 of them may take part in the group event. Teams of less than 4 members will not be allowed to participate.
3. Members of each team, **accompanied by the teacher-in-charge, should wear proper school uniform** and present **ID Card or student identification document** when registering at the venue reception **not later than 9:00 a.m.** Failing to do so, the team **will be disqualified.**
4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, instructions written in both Chinese and English will be provided. Question papers are printed in both English and Chinese.
5. Each individual event consists of 4 parts. Each part must be completed by one member of the team. Help from other team members would result in disqualification for that particular event.
6. In an individual event, the four parts are interrelated. When solving Part 2, one has to make use of the answer obtained in Part 1, and so on.
7. In a group event, the four parts are to be done by the whole team and the parts may or may not be interrelated. Discussions are allowed provided that voice level is kept to a minimum.
8. Use of calculating devices will not be allowed; otherwise the team will risk disqualification or deduction of marks.
9. Participants having electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) or any alarm device(s), should turned them off and put them inside their bags or under their chairs. Failing to do so, the team **will risk disqualification.**
10. All answers should be numerical and reduced to the simplest form unless stated otherwise. No proof or working is required.
11. The time limit for each event is 5 minutes.
12. The Marking System is as follows:
 - (a) Scores for accuracy:

| <u>Individual Events</u> | <u>Scores</u> | <u>Group Events</u> | <u>Scores</u> |
|--------------------------|---------------|---------------------|---------------|
| Part 1 correct ... | 1 | Any 1 part correct | ...2 |
| Part 2 correct ... | 2 | Any 2 parts correct | ...4 |
| Part 3 correct ... | 3 | Any 3 parts correct | ...7 |
| Part 4 correct ... | 4 | All 4 parts correct | ...10 |
| Total | 10 | | |
 - (b) Multiplying factors for speed:

| | |
|--|----------|
| <i>Time taken for the teams to hand in their answer < 1 min.</i> | 4 |
| <i>1 min. ≤ Time taken for the teams to hand in their answer < 2 min.</i> | 3 |
| <i>2 min. ≤ Time taken for the teams to hand in their answer < 3 min.</i> | 2 |
| <i>Time taken for the teams to hand in their answer ≥ 3 min.</i> | 1 |
 - (c) Bonus Score:

Teams, which hand in their answers of anyone individual/group event have all the answers in that event correct, will be awarded a bonus score of 20 marks.
 - (d) Total score for each event:

(Score for accuracy) × (Multiplying factor) + (Bonus score)
13. Any queries should reach the Judging Panel within 10 minutes after the end of the last individual group event. The decision of the Judging Panel on the queries is final.
14. Trophies and prizes will be given to the three schools achieving the highest scores. The champion school may keep the Champion shield until the next Hong Kong Mathematics Olympiad.
15. The decision of the Judging Panel on the overall results is final.

比賽資料 決賽隊伍數目：50 決賽日期：2019 年 4 月 27 日星期六 地點：香港教育大學
決賽名單：

| School ID | Name of School <small>school = new school entering final event this year</small> |
|-----------|--|
| FE-01 | Baptist Lui Ming Choi Secondary School |
| FE-02 | Bishop Hall Jubilee School |
| FE-03 | Carmel Divine Grace Foundation Secondary School |
| FE-04 | Cheung Chuk Shan College |
| FE-05 | Cheung Sha Wan Catholic Secondary School |
| FE-06 | Christian Alliance S.C. Chan Memorial College |
| FE-07 | Diocesan Boys' School |
| FE-08 | Diocesan Girls' School |
| FE-09 | FDBWA Szeto Ho Secondary School |
| FE-10 | Fukien Secondary School |
| FE-11 | Good Hope School |
| FE-12 | Heung To Middle School |
| FE-13 | HKSYC & IA Wong Tai Shan Memorial College |
| FE-14 | HKTA Tang Hin Memorial Secondary School |
| FE-15 | Hong Kong Baptist University Affiliated School Wong Kam Fai Secondary and Primary School |
| FE-16 | Hong Kong Chinese Women's Club College |
| FE-17 | Kiangsu-Chekiang College (Shatin) |
| FE-18 | King Ling College |
| FE-19 | La Salle College |
| FE-20 | Munsang College |
| FE-21 | NTHYK Yuen Long District Secondary School |
| FE-22 | Po Leung Kuk Celine Ho Yam Tong College |
| FE-23 | Po Leung Kuk Centenary Li Shiu Chung Memorial College |
| FE-24 | Po Leung Kuk No. 1 WH Cheung College |
| FE-25 | Po Leung Kuk Tang Yuk Tien College |
| FE-26 | Po On Commerce Association Wong Siu Ching Secondary School |
| FE-27 | Pui Ching Middle School |
| FE-28 | Queen Elizabeth School |
| FE-29 | Queen's College |
| FE-30 | Sha Tin Government Secondary School |
| FE-31 | Sing Yin Secondary School |
| FE-32 | SKH Bishop Mok Sau Tseng Secondary School |
| FE-33 | SKH Lam Woo Memorial Secondary School |
| FE-34 | SKH Tang Shiu Kin Memorial Secondary School |
| FE-35 | SKH Tsang Shiu Tim Memorial Secondary School |
| FE-36 | St Joseph's College |
| FE-37 | St Mark's School |
| FE-38 | St Paul's Co-Educational College |
| FE-39 | St Paul's College |
| FE-40 | STFA Lee Shau Kee College |
| FE-41 | The Chinese Foundation Secondary School |
| FE-42 | The ELCHK Yuen Long Lutheran Secondary School |
| FE-43 | Tsuen Wan Government Secondary School |
| FE-44 | TWGH Lo Kon Ting Memorial College |
| FE-45 | TWGH Li Ka Shing College |
| FE-46 | Wa Ying College |
| FE-47 | Wah Yan College, Hong Kong |
| FE-48 | Wah Yan College, Kowloon |
| FE-49 | Ying Wa College |
| FE-50 | Yuen Long Merchants Association Secondary School |

Hong Kong Mathematics Olympiad (2018 – 2019)

Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 A 是 $(x^2 + 2)^5$ 展開式中 x^4 的係數，求 A 的值。

If A is the coefficient of x^4 in the expansion of $(x^2 + 2)^5$, determine the value of A .

$A =$

2. 若 x 和 y 為正整數，並且滿足以下等式

$$\log_{10} x + \log_{10} y = \log_{10} (2x - Ay) + 1,$$

而 B 是 (x, y) 所有可能組合的數量，求 B 的值。

If x and y are positive integers that satisfy

$$\log_{10} x + \log_{10} y = \log_{10} (2x - Ay) + 1,$$

and B is the number of possible pairs of (x, y) , determine the value of B .

$B =$

3. 若 $Y = 2^{3(B-1)}$ 並且 C 是 Y 中每個數字之和，求 C 的值。

If $Y = 2^{3(B-1)}$ and C is the sum of the digits of Y , determine the value of C .

$C =$

4. 在 $\triangle XYZ$ 中，已知 $XY \perp YZ$, $\angle XZY = \theta$, 綫段 YZ 和 XZ 的長度分別為 $C - 3$ 和 $C + 5$ 。若 $D = (\sin \theta + \tan \theta)^2$, 求 D 的值。

In $\triangle XYZ$, $XY \perp YZ$, $\angle XZY = \theta$, and the length of YZ and XZ are $C - 3$ and $C + 5$ respectively. If $D = (\sin \theta + \tan \theta)^2$, determine the value of D .

$D =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+ Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (2018 – 2019)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $\sqrt{A} = \sqrt{11 + \sqrt{21}} - \sqrt{11 - \sqrt{21}}$ ，求 A 的值。

If $\sqrt{A} = \sqrt{11 + \sqrt{21}} - \sqrt{11 - \sqrt{21}}$, determine the value of A .

$A =$

2. 若直綫 $y = mx + B$ 經過兩點 $(4, 5)$ 和 $(-A, A)$ ，求 B 的值。

If the straight line $y = mx + B$ passes through the two points $(4, 5)$ and $(-A, A)$, determine the value of B .

$B =$

3. 若 $\cos x + \sin x = \frac{2B}{5}$ 及 $C = (\tan x + \cot x)^{-1}$ ，求 C 的值。

If $\cos x + \sin x = \frac{2B}{5}$ and $C = (\tan x + \cot x)^{-1}$, determine the value of C .

$C =$

4. 假設 D, x, y 和 z 均為整數，其中 $D > x > y > z$ 。

若 D, x, y 和 z 滿足等式 $3^D - 3^x + 3^y - 3^z = \frac{1000C + 2}{9}$ ，求 D 的值。

Suppose that D, x, y and z are integers with $D > x > y > z$. If D, x, y and z satisfy the equation $3^D - 3^x + 3^y - 3^z = \frac{1000C + 2}{9}$, determine the value of D .

$D =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+ Bonus
score

Time

Total score

Min.

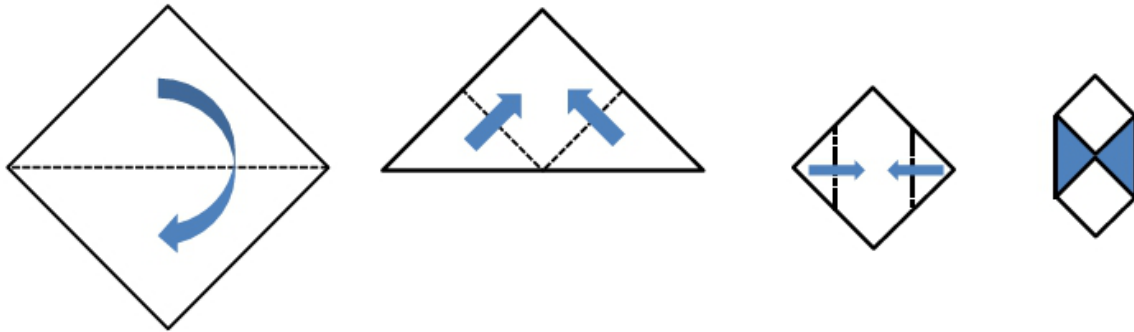
Sec.

Hong Kong Mathematics Olympiad (2018 – 2019)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 一張正方形紙的面積為 100 cm^2 ，按照圖中的虛線和箭咀的方向對摺。
 若圖中的陰影部份為 $s \text{ cm}^2$ ，求 s 的值。

A piece of **square** paper of area 100 cm^2 , is folded in half along the dotted line as shown below. If the area of the shaded region in the last figure is $s \text{ cm}^2$, determine the value of s .



2. 假設 $s = \frac{A}{B}$ 為最簡分數。若 c 和 d 分別為 A 和 B 的正因數的數量，
 求 $t = c + d$ 的值。

Suppose that $s = \frac{A}{B}$ is in simplest form. If c and d are the number of positive factors of A and B respectively, determine the value of $t = c + d$.

3. 若今天是某一週的第 2 日，以及已知 t^{2019} 日後的當天為該週的第 u 日，求 u 的值。
 If today is the 2nd day of a week, and **it is** known that t^{2019} days later is the u^{th} day of the week, determine the value of u .

4. 若 $v_{(u+5)}$ 為 $1231234_{(u+5)} \div 123_{(u+5)}$ 的餘數，求 v 的值。

If $v_{(u+5)}$ is the remainder of $1231234_{(u+5)} \div 123_{(u+5)}$, determine the value of v .

FOR OFFICIAL USE

Score for accuracy

×

Mult. factor for speed

=

Team No.

+ Bonus score

Time

Total score

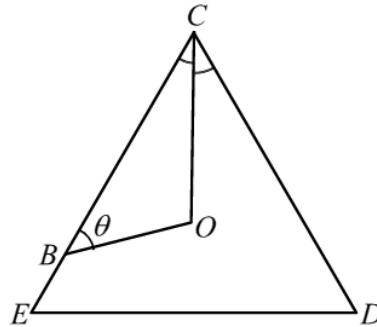
Min.

Sec.

Hong Kong Mathematics Olympiad (2018 – 2019)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

1. $\triangle CDE$ 為一個等邊三角形。點 O 在 $\triangle CDE$ 內。若點 B 在 CE 上， $\theta = \angle CBO$ ， OC 為 $\angle DCE$ 的角平分綫，以及 $OC : OB = 5 : 4$ ，求 $\alpha = \sin \theta$ 的值。
- $\triangle CDE$ is an equilateral triangle. Point O is inside $\triangle CDE$. If point B lies on CE , $\theta = \angle CBO$, $\angle DCE$ is bisected by OC , and $OC : OB = 5 : 4$, determine the value of $\alpha = \sin \theta$.



$\alpha =$

2. 假設有一函數 $f(x)$ ，對於任何整數 x 及任何整數 $y \neq 0$ ，均滿足 $f\left(\frac{x}{y}\right) = f(x) - f(y)$ 和 $f(2) = -1$ 。若 $\beta = f\left(\frac{\alpha}{10}\right)$ ，求 β 的值。

$\beta =$

Suppose that there exists a function $f(x)$, defined for all integers x and for all integers $y \neq 0$, such that $f\left(\frac{x}{y}\right) = f(x) - f(y)$ and $f(2) = -1$. If $\beta = f\left(\frac{\alpha}{10}\right)$, determine the value of β .

3. 若 $B = \gamma p + 2\gamma(1 - p)$ 以及 $p = \frac{\beta(\gamma - 40)}{100}$ ，當 B 取最大值時，求 γ 的值。

If $B = \gamma p + 2\gamma(1 - p)$ and $p = \frac{\beta(\gamma - 40)}{100}$, determine the value of γ such that B attains the maximum.

$\gamma =$

4. 已知 a 、 b 和 c 是 γ 的正因數，而且 $a < b < c < \gamma$ 及 $ab = c$ 。若 $x + y = a$ ， $x + 2y + z = b$ ， $y + 2z + t = c$ 及 $\delta = x + y + z + t$ ，求 δ 的值。
- Given that a , b and c are positive factors of γ with $a < b < c < \gamma$ and $ab = c$. If $x + y = a$, $x + 2y + z = b$, $y + 2z + t = c$ and $\delta = x + y + z + t$, determine the value of δ .

$\delta =$

FOR OFFICIAL USE

Score for accuracy

\times

Mult. factor for speed

$=$

Team No.

+ Bonus score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (2018–2019)
Final Event 1 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

1. 已知 $x + y = 32$ ，其中 $x, y \geq 0$ 。若 a 為 $\sqrt{x} + \sqrt{y}$ 的最大值，求 a 的值。

Let $x + y = 32$ with $x, y \geq 0$. If a is the maximum value of $\sqrt{x} + \sqrt{y}$, determine the value of a .

2. 一個盒中只有 x 個一元硬幣， $x + 2$ 個二元硬幣及 $x + 4$ 個五元硬幣。已知隨機從盒中拿出一元硬幣的概率小於 0.1。若盒中有 b 個硬幣，求 b 的值。

A box contains only x -one-dollar coins, $x + 2$ two-dollar coins and $x + 4$ five-dollar coins. Given that the probability of drawing a one-dollar coin randomly from the box is less than 0.1. If the box contains b coins, determine the value of b .

3. 若 c 是以下數的最大公因數，

$$3^3 - 3, 5^5 - 5, 7^7 - 7, 9^9 - 9, \dots, 2019^{2019} - 2019,$$

求 c 的值。

If c is the greatest common factor of the following numbers

$$3^3 - 3, 5^5 - 5, 7^7 - 7, 9^9 - 9, \dots, 2019^{2019} - 2019,$$

determine the value of c .

4. 設 $x = \frac{\sqrt{5} + \sqrt{7}}{\sqrt{5} - \sqrt{7}}$ 和 $y = \frac{\sqrt{5} - \sqrt{7}}{\sqrt{5} + \sqrt{7}}$ 。若 $d = 3x^2 - 7xy + 3y^2$ ，求 d 的值。

Let $x = \frac{\sqrt{5} + \sqrt{7}}{\sqrt{5} - \sqrt{7}}$ and $y = \frac{\sqrt{5} - \sqrt{7}}{\sqrt{5} + \sqrt{7}}$. If $d = 3x^2 - 7xy + 3y^2$, determine the value of d .

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+
Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (2018 – 2019)
Final Event 2 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

1. 設 $X = \sqrt{2020 - \sqrt{A}}$ 為正整數，求 A 的最小值。

Let $X = \sqrt{2020 - \sqrt{A}}$ be a positive integer. Determine the least value of A .

2. 假設 $\begin{cases} x + y = 5 \\ 4x^2 + y^2 = 80 \end{cases}$ ，及 $P = (x_1, y_1)$ 和 $Q = (x_2, y_2)$ 為兩個不同的點，同時滿足這兩個等式。若 $B = y_1 - x_1 + y_2 - x_2$ ，求 B 的值。

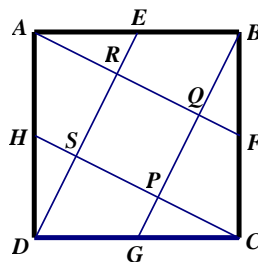
Suppose that $\begin{cases} x + y = 5 \\ 4x^2 + y^2 = 80 \end{cases}$, and $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are two different points, simultaneously satisfy these two equations. If $B = y_1 - x_1 + y_2 - x_2$, determine the value of B .

3. 若 $Q = a^b - b^a$ 為正整數，求 Q 的最小值。

If $Q = a^b - b^a$ is a positive integer, determine the least value of Q .

4. 在正方形 $ABCD$ 中， E 、 F 、 G 和 H 分別是 AB 、 BC 、 CD 和 AD 的中點。 DE 分別與 AF 和 CH 相交於點 R 和 S 。 BG 分別與 AF 和 CH 相交於點 Q 和 P 。若 U 是正方形 $ABCD$ 的面積，而 V 是四邊形 $PQRS$ 的面積，求 $W = \frac{U}{V}$ 的值。

In square $ABCD$, E, F, G, H are the mid-points of AB, BC, CD and AD respectively. DE intersects with AF and CH at R and S respectively. Moreover, BG intersects with AF and CH at Q and P respectively. If U is the area of square $ABCD$ and V is the area of the quadrilateral $PQRS$, determine the value of $W = \frac{U}{V}$.



FOR OFFICIAL USE

Score for accuracy

×

Mult. factor for speed

=

Team No.

+ Bonus score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (2018 – 2019)

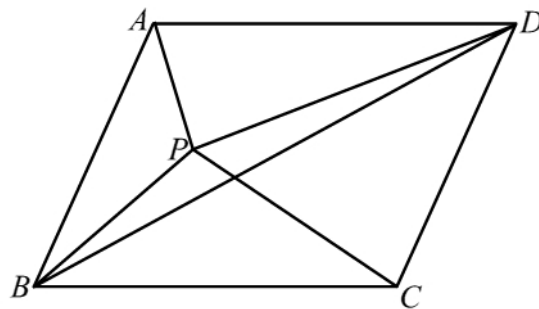
Final Event 3 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 若 $\sqrt{32 \times 81 \times 343} = b\sqrt{a}$ ，其中 a 和 b 是正整數，求 a 的最小值。
If $\sqrt{32 \times 81 \times 343} = b\sqrt{a}$, where a and b are positive integers, determine the least value of a .

2. 下圖中， P 點在平行四邊形 $ABCD$ 內。若 $\triangle ABP$ 、 $\triangle BPC$ 和 $\triangle BPD$ 的面積分別為 73 cm^2 、 100 cm^2 和 $e \text{ cm}^2$ ，求 e 的值。
In the diagram below, point P is inside parallelogram $ABCD$. If areas of $\triangle ABP$, $\triangle BPC$ and $\triangle BPD$ are 73 cm^2 , 100 cm^2 and $e \text{ cm}^2$ respectively, determine the value of e .



3. 在以下的 3×3 幻方中，每行、列和兩斜行(對角線)的和相等。如下圖所示，部份數值已經填上。求 c 的值。

A 3×3 magic square is filled with a number in each square such that the sum of the three numbers in each row, column and the two main diagonals are equal. The partially completed grid is shown below. Determine the value of c .

| | | |
|-----|----|----|
| c | 16 | 20 |
| 2 | | |
| | | |

4. 若 $X = 2^{2018} + 3^{2018}$ 及 d 是其個位數字，求 d 的值。
If $X = 2^{2018} + 3^{2018}$ and d is the units digit, determine the value of d .

FOR OFFICIAL USE

| | | | | | | |
|--------------------|----------------------|----------|------------------------|----------------------|-----|----------------------|
| Score for accuracy | <input type="text"/> | \times | Mult. factor for speed | <input type="text"/> | $=$ | <input type="text"/> |
| | | | + | Bonus score | | <input type="text"/> |
| | | | | Total score | | <input type="text"/> |

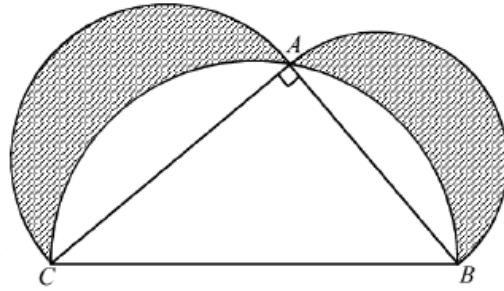
| | |
|----------|----------------------|
| Team No. | <input type="text"/> |
| Time | <input type="text"/> |
| | <input type="text"/> |
| Min. | Sec. |

Hong Kong Mathematics Olympiad (2018 – 2019)
Final Event 4 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

1. 如圖所示， $\triangle ABC$ 是一直角三角形，其中 $AC = 8$ ， $BC = 10$ 。以 AB 、 AC 和 BC 為直徑分別畫了三個半圓。若陰影部分的總面積是 α ，求 α 的值。



$\alpha =$

As shown in the figure, $\triangle ABC$ is a right-angled triangle with $AC = 8$ and $BC = 10$. Semi-circles are drawn with AB , AC and BC as diameters. If the total shaded area is α , determine the value of α .

2. 對所有的正整數 n ，設某一個函數 $F(n)$ 有如下定義：

$F(1) = 0$ ，

當 $n \geq 2$ ，

如果 n 只能被 2 整除而不能被 3 整除，則 $F(n) = F(n-1) + 2$ ；

如果 n 只能被 3 整除而不能被 2 整除，則 $F(n) = F(n-1) + 3$ ；

如果 n 既能被 2 整除而又能被 3 整除，則 $F(n) = F(n-1) + 4$ ；

如果 n 既不能被 2 整除而又不能被 3 整除，則 $F(n) = F(n-1)$ 。

若 $\beta = F(4000)$ ，求 β 的值。

For all positive integers n , suppose there exists a function $F(n)$ defined as follows:

$F(1) = 0$,

for all $n \geq 2$,

$F(n) = F(n-1) + 2$ if 2 divides n but 3 does not divide n ;

$F(n) = F(n-1) + 3$ if 3 divides n but 2 does not divide n ;

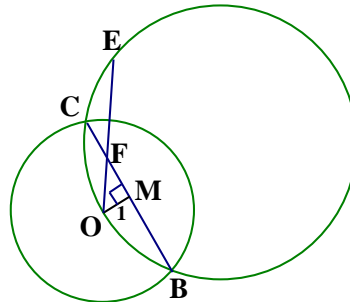
$F(n) = F(n-1) + 4$ if 2 and 3 both divide n ;

$F(n) = F(n-1)$ if neither 2 nor 3 divides n .

If $\beta = F(4000)$, determine the value of β .

3. 如圖所示，兩圓相交於 B 、 C 兩點。 M 是 BC 的中點。 O 在大圓上，使得 $OM \perp BC$ 。 $OM = 1$ 、 $OC = 3$ 及 $OE = 5$ 。若 $\gamma = OF$ ，求 γ 的值。

Two circles intersect at B , C as shown in the figure. M is the mid-point of BC . O is a point on the larger circle, so that $OM \perp BC$. $OM = 1$, $OC = 3$ and $OE = 5$. If $\gamma = OF$, determine the value of γ .



$\gamma =$

4. 若 $f(x) = \left(x + \frac{1}{2000}\right) \times \left(x + \frac{1}{2001}\right) \times \cdots \times \left(x + \frac{1}{2019}\right)$ 以及 $\delta = f(1) - 1$ ，求 δ 的值。

If $f(x) = \left(x + \frac{1}{2000}\right) \times \left(x + \frac{1}{2001}\right) \times \cdots \times \left(x + \frac{1}{2019}\right)$ and $\delta = f(1) - 1$,

determine the value of δ .

$\delta =$

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Score for
accuracy

\times

Mult. factor for
speed

$=$

Team No.

+

Bonus
score

Time

Total score

Min.

Sec.

成績 Results

| School code | School Name | Ind | | | | Isum Group | | | | Gsum Total rank | | | |
|-------------|--|---------|---------|---------|---------|------------|---------|---------|---------|-----------------|---------|---------|---------|
| | | Event 1 | Event 2 | Event 3 | Event 4 | Event 1 | Event 2 | Event 3 | Event 4 | Event 1 | Event 2 | Event 3 | Event 4 |
| FE-01 | Baptist Lui Ming Choi Secondary School | 4 | 0 | 7 | 0 | 11 | 4 | 7 | | 11 | 22 | 28 | |
| FE-02 | Bishop Hall Jubilee School | 0 | 1 | 4 | 0 | 5 | 7 | 2 | | 9 | 14 | 40 | |
| FE-03 | Carmel Divine Grace Foundation Secondary School | 0 | 0 | 12 | 1 | 13 | 8 | 8 | | 16 | 29 | 20 | |
| FE-04 | Cheung Chuk Shan College | 1 | 1 | 0 | 1 | 3 | 2 | 4 | | 6 | 9 | 46 | |
| FE-05 | Cheung Sha Wan Catholic Secondary School | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 48 | |
| FE-06 | Christian Alliance S.C. Chan Memorial College | 1 | 2 | 12 | 2 | 17 | 8 | 7 | | 15 | 32 | 17 | |
| FE-07 | Diocesan Boys' School | 1 | 6 | 40 | 1 | 48 | 4 | 30 | | 34 | 82 | 5 | |
| FE-08 | Diocesan Girls' School | 6 | 6 | 30 | 0 | 42 | 4 | 40 | | 44 | 86 | 3 | |
| FE-09 | FDBWA Szeto Ho Secondary School | 3 | 3 | 0 | 1 | 7 | 4 | 8 | | 12 | 19 | 33 | |
| FE-10 | Fukien Secondary School | 1 | 6 | 7 | 3 | 17 | 2 | 7 | | 9 | 26 | 24 | |
| FE-11 | Good Hope School | 1 | 0 | 3 | 0 | 4 | 7 | 2 | | 9 | 13 | 42 | |
| FE-12 | Heung To Middle School | 2 | 0 | 5 | 0 | 7 | 7 | 7 | | 14 | 21 | 29 | |
| FE-13 | HKSYC & IA Wong Tai Shan Memorial College | 1 | 6 | 4 | 1 | 12 | 4 | 7 | | 11 | 23 | 25 | |
| FE-14 | HKTA Tang Hin Memorial Secondary School | 1 | 0 | 3 | 0 | 4 | 8 | 8 | | 16 | 20 | 31 | |
| FE-15 | Hong Kong Baptist University Affiliated School Wong Kam Fai Secondary and Primary School | 4 | 6 | 6 | 1 | 17 | 7 | 7 | | 14 | 31 | 18 | |
| FE-16 | Hong Kong Chinese Women's Club College | 0 | 1 | 4 | 0 | 5 | 7 | 7 | | 14 | 19 | 33 | |
| FE-17 | Kiangsu-Chekiang College (Shatin) | 0 | 0 | 12 | 0 | 12 | 8 | 7 | | 15 | 27 | 22 | |
| FE-18 | King Ling College | 0 | 0 | 3 | 15 | 18 | 7 | 8 | | 15 | 33 | 16 | |
| FE-19 | La Salle College | 30 | 6 | 40 | 3 | 79 | 30 | 40 | | 70 | 149 | 1 | |
| FE-20 | Munsang College | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | 0 | 0 | 48 | |
| FE-21 | NTHYK Yuen Long District Secondary School | 1 | 30 | 0 | 0 | 31 | 4 | 7 | | 11 | 42 | 13 | |
| FE-22 | Po Leung Kuk Celine Ho Yam Tong College | 0 | 0 | 4 | 0 | 4 | 7 | 4 | | 11 | 15 | 38 | |
| FE-23 | Po Leung Kuk Centenary Li Shiu Chung Memorial College | 1 | 0 | 30 | 1 | 32 | 30 | 30 | | 60 | 92 | 2 | |
| FE-24 | Po Leung Kuk No. 1 WH Cheung College | 1 | 6 | 6 | 1 | 14 | 40 | 30 | | 70 | 84 | 4 | |
| FE-25 | Po Leung Kuk Tang Yuk Tien College | 1 | 4 | 4 | 0 | 9 | 4 | 7 | | 11 | 20 | 31 | |
| FE-26 | Po On Commerce Association Wong Siu Ching Secondary School | 1 | 0 | 0 | 0 | 1 | 7 | 8 | | 15 | 16 | 37 | |
| FE-27 | Pui Ching Middle School | 6 | 3 | 0 | 1 | 10 | 30 | 40 | | 70 | 80 | 6 | |

成績 Results

| | | | | | | | | | | | | | |
|-------|--|---|---|----|---|----|----|----|--|--|----|----|----|
| FE-28 | Queen Elizabeth School | 0 | 3 | 0 | 1 | 4 | 2 | 2 | | | 4 | 8 | 47 |
| FE-29 | Queen's College | 1 | 3 | 30 | 1 | 35 | 4 | 30 | | | 34 | 69 | 8 |
| FE-30 | Sha Tin Government Secondary School | 1 | 3 | 0 | 1 | 5 | 4 | 2 | | | 6 | 11 | 43 |
| FE-31 | Sing Yin Secondary School | 3 | 3 | 7 | 1 | 14 | 30 | 8 | | | 38 | 52 | 10 |
| FE-32 | SKH Bishop Mok Sau Tseng Secondary School | 6 | 0 | 9 | 0 | 15 | 4 | 4 | | | 8 | 23 | 25 |
| FE-33 | SKH Lam Woo Memorial Secondary School | 0 | 6 | 5 | 1 | 12 | 7 | 4 | | | 11 | 23 | 25 |
| FE-34 | SKH Tang Shiu Kin Memorial Secondary School | 1 | 4 | 2 | 0 | 7 | 14 | 7 | | | 21 | 28 | 21 |
| FE-35 | SKH Tsang Shiu Tim Memorial Secondary School | 1 | 5 | 30 | 1 | 37 | 4 | 7 | | | 11 | 48 | 12 |
| FE-36 | St Joseph's College | 1 | 3 | 40 | 1 | 45 | 7 | 4 | | | 11 | 56 | 9 |
| FE-37 | St Mark's School | 0 | 0 | 30 | 1 | 31 | 2 | 4 | | | 6 | 37 | 15 |
| FE-38 | St Paul's Co-Educational College | 1 | 3 | 5 | 1 | 10 | 2 | 30 | | | 32 | 42 | 13 |
| FE-39 | St Paul's College | 1 | 3 | 7 | 1 | 12 | 2 | 7 | | | 9 | 21 | 29 |
| FE-40 | STFA Lee Shau Kee College | 0 | 0 | 6 | 0 | 6 | 2 | 2 | | | 4 | 10 | 44 |
| FE-41 | The Chinese Foundation Secondary School | 0 | 0 | 4 | 0 | 4 | 2 | 4 | | | 6 | 10 | 44 |
| FE-42 | The ELCHK Yuen Long Lutheran Secondary School | 1 | 6 | 4 | 0 | 11 | 4 | 2 | | | 6 | 17 | 35 |
| FE-43 | Tsuen Wan Government Secondary School | 1 | 4 | 9 | 0 | 14 | 6 | 7 | | | 13 | 27 | 22 |
| FE-44 | TWGH Lo Kon Ting Memorial College | 1 | 7 | 4 | 2 | 14 | 30 | 7 | | | 37 | 51 | 11 |
| FE-45 | TWGH Li Ka Shing College | 0 | 0 | 4 | 0 | 4 | 8 | 2 | | | 10 | 14 | 40 |
| FE-46 | Wa Ying College | 3 | 2 | 12 | 1 | 18 | 8 | 4 | | | 12 | 30 | 19 |
| FE-47 | Wah Yan College, Hong Kong | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0 | 0 | 48 |
| FE-48 | Wah Yan College, Kowloon | 0 | 0 | 8 | 0 | 8 | 2 | 7 | | | 9 | 17 | 35 |
| FE-49 | Ying Wa College | 1 | 7 | 5 | 0 | 13 | 30 | 30 | | | 60 | 73 | 7 |
| FE-50 | Yuen Long Merchants Association Secondary School | 1 | 0 | 0 | 0 | 1 | 7 | 7 | | | 14 | 15 | 38 |

| | |
|---------------|-------------------------|
| Champion | La Salle College |
| 1st runner up | Diocesan Boys' School |
| 2nd runner up | Pui Ching Middle School |