第四十一屆香港數學競賽(2023/24)
初賽規則

1. 初賽分個人項目和團體項目兩部分。個人項目限時**六十分鐘**，團體項目限時**二十分鐘**。

2. 每間學校可提名4至6位**中五**或以下同學參賽。其中任何4位可參加個人項目；又其中任何4位可參加團體項目。不足4位同學的隊伍將被撤銷參賽資格。

3. 所有參賽學生**必須穿著整齊校服**，**並由負責教師帶領**，初賽將會準時於上午9時開始。

4. 初賽題目以中、英文並列。指示語言將採用粵語。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英文並列。

5. 每一隊員於個人項目中須解答15條問題（當中甲部佔**10**題、乙部佔**5**題）；而每一隊員則於團體項目中須解答10條問題（當中甲部佔**5**題、乙部佔**5**題）。

6. 團體項目中，各參賽隊員可進行討論，但必須將聲浪降至最低。

7. 初賽時，**不准使用**計算機、四位對數表、量角器、圓規、三角尺及直尺等工具，違例學生將被撤銷參賽資格或扣分。

8. **除非另有聲明**，**否則所有問題的答案均為數字**，**並應化至最簡**，**但無須呈交證明及算草**。

9. 參賽者須關掉所有電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，否則大會有權**取消該學生參賽資格**。

10. 個人項目中，甲部和乙部的每一正確答案分別可得1分及2分。每隊可得之最高總積分為80分。

11. 團體項目中，甲部和乙部的每一正確答案分別可得2分及3分。每隊可得之最高總積分為25分。

12. 初賽中，並不給予快捷分。

13. 參賽者必須自備書寫工具，例如: **原子筆**及**鉛筆**。

14. 籌委會將根據各參賽隊伍的總成績（個人項目及團體項目的積分總和）選出最高積分的五十隊進入決賽。

15. 獎項：

    (a) 於個人項目和團體項目中，根據參賽者所得分數由高至低排列後

        (i) 取得滿分者將獲頒予最佳表現及積分獎狀；

        (ii) 除上述(i) 中取得最佳表現的參賽者外，

            (1) 成績最佳的首2%參賽者將獲頒予一等榮譽獎狀；

            (2) 隨後的5%參賽者將獲頒予二等榮譽獎狀；

            (3) 隨後的10%參賽者將獲頒予三等榮譽獎狀。

            (4) 隨後的13%參賽者將獲頒予優秀表現獎狀。

    (b) 總成績（個人項目及團體項目的積分總和）於各分區（即港島、九龍一區、九龍二區、新界東及新界西）最高之首10% 的參賽隊伍將獲頒予獎狀。

16. 如有任何疑問，參賽者須於初賽完畢後，立即透過負責教師致電2153 7436向籌委會的教育局代表鄭仕文先生提出。所提出之疑問，將由籌委會作最後裁決。

The Forty First Hong Kong Mathematics Olympiad (2023/24)
Regulations (Heat Events)

1. The Heats consists of two parts, namely, individual and Group Events. Individual Event will last for **60 minutes** and Group Event will last for **20 minutes**.
2. Each school may nominate **4 to 6 student participants** of **Secondary 5 level or below**. Any 4 of them may take part in the Individual Event and any 4 of them may take part in the Group Event. Teams of less than 4 members will be disqualified.
3. All student participants, **accompanied by the teacher-in-charge, should wear proper school uniform**. The competition will commence at 9:00 a.m. sharp.
4. Question papers are printed in both Chinese and English. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, written instructions in both Chinese and English will be provided. Question papers are printed in both Chinese and English.
5. Each participant has to solve 15 questions in the Individual Event (***10 questions in Part A*** and ***5 questions in Part B***), and 10 questions in the Group Event (***5 questions in Part A*** and ***5 questions in Part B***).
6. In the Group Event, discussions among participating team members are allowed provided that the voice level is kept to a minimum.
7. Devices such as calculators, four-figure tables, protractors, compasses, set squares and rulers will **NOT** be allowed to use throughout the Heats, otherwise the participant will be disqualified or risk deduction of marks.
8. **All answers in the Individual and the Group Events should be numerical and reduced to the simplest form unless stated otherwise. No proof or demonstration of work is required.**
9. All electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) and alarm device(s) should be turned off during the Heats. Failing to do so, the participant **will risk disqualification**.
10. For the Individual Event, 1 mark and 2 marks will be given to each correct answer in Part A and Part B respectively. The total maximum score for a school team should be 80.
11. For the Group Event, 2 marks and 3 marks will be given to each correct answer in Part A and Part B respectively. The total maximum score for a school team should be 25.
12. No mark for speed will be awarded in the Heats.
13. Participants should bring along their own writing instruments, e.g. **ball pens** and **pencils**.
14. Based on the highest aggregate scores (sum of the scores in the Individual and the Group Events), the Organising Committee will select the 50 highest scored teams entering the Finals.
15. Awards:
    (a) For each of Individual and Group Events, after arranging all participant's scores in the order from the highest to the lowest
        (i) participants obtaining full score will be awarded Best Performance honour certificates ;
        (ii) after deducting those participants with full score achievement as in (i),
            (1) the first 2% from the remaining participants in the said order will be awarded First-class Honour certificates ;
            (2) the next 5% from the remaining participants will be awarded Second-class Honour certificates ; and
            (3) the next 10% from the remaining participants will be awarded Third-class Honour certificates ;
            (4) the next 13% from the remaining participants will be awarded Honourable Mention certificates ;
    (b) About 10% of participating schools with the highest aggregate scores (sum of the scores in the Individual and Group Events) in each of the regions (Hong Kong Island, Kowloon Region One, Kowloon Region Two, the New Territories East, and the New Territories West) will be awarded certificates of merit.
16. Should there be any queries, participants should reach Mr CHENG Sze-man, the representative of EDB in the Organising Committee, **via the teacher-in-charge** at 2153 7436 immediately after the Heats. The decision of the Organising Committee on the queries is final.

Hong Kong Mathematics Olympiad (2023-2024)
Heats – Individual Event
香港數學競賽 (2023 – 2024)
初賽個人項目

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。第一至十題，每題 1 分，第十一至十五題，每題 2 分。全卷滿分 20 分。

時限：1 小時

Unless otherwise stated, all answers should be given in exact numerals in their simplest form. No approximation is accepted. The diagrams are not necessarily drawn to scale. Q1- Q10 1 mark each, Q11-Q15 2 marks each. The maximum mark for this paper is 20. Time allowed: 1 hour

1. 求 2024 的所有因數之和。
   Find the sum of the factors of 2024.

2. 若 $a^{3y} = 729$，求 a^{-2y} 的值。
   If $a^{3y} = 729$, find the value of a^{-2y}.

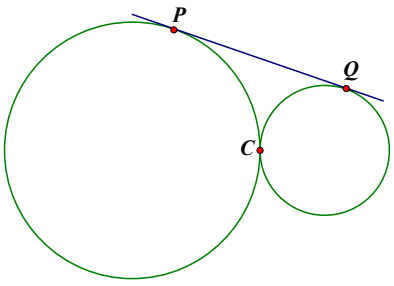
3. 一個 6 位數由兩個相同的 3 位數組合而成，如 256256 及 678678。求這些 6 位數的最大公因數。
   A 6-digit number is formed by joining two identical 3-digit numbers, such as 256256 and 678678. Find the greatest common factor of these 6-digit numbers.

4. 若 $4^{x+3} - 47 = 193 + 4^{x+1}$，求 $\left(4^{x+3}\right)\left(4^{x+1}\right)$ 的值。
   If $4^{x+3} - 47 = 193 + 4^{x+1}$, find the value of $\left(4^{x+3}\right)\left(4^{x+1}\right)$.

5. 在一直角三角形中，從銳角頂點所作的中線長度為 7 及 9。求三角形斜邊的長。
   In a right-angled triangle, the lengths of the medians from the vertices of the acute angles are 7 and 9. Find the length of the hypotenuse of the triangle.

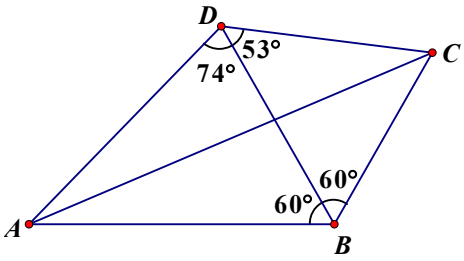
6. 志偉生於 20 世紀 (1901–2000)，於 y^2 年時的歲數為 y。求志偉的出生年份。
   Eric was born in 20th century (1901–2000), and he was y years old in the year of y^2. Find his year of birth.

7. 如圖一所示，一條公切線與一大圓及一小圓分別相交於點 P 及 Q。已知該兩圓相交於點 C 且它們的半徑分別為 49 及 25，求 PQ 的長。
   As shown in Figure 1, a common tangent touches a large circle and a small circle at P and Q respectively. Given that the two circles touch each other at C and their radii are 49 and 25 respectively, find the length of PQ.



圖一 Figure 1

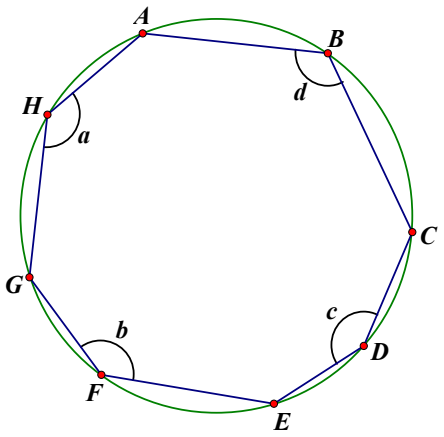
8. 如圖二所示，$ABCD$ 是一個四邊形。若 $\angle ABD = \angle CBD = 60°$，$\angle ADB = 74°$ 及 $\angle CDB = 53°$，求 $\angle BAC$ 的值。
   As shown in Figure 2, $ABCD$ is a quadrilateral. If $\angle ABD = \angle CBD = 60°$, $\angle ADB = 74°$ and $\angle CDB = 53°$, find the value of $\angle BAC$.



圖二 Figure 2

9. 設 a 為正實數。若方程組 $\begin{cases}(a+3)x+(a+2)y=1\\(a-1)x-ay=1\end{cases}$ 無解，求 a 的值。
   Let a be a positive real number. If the system of equations $\begin{cases}(a+3)x+(a+2)y=1\\(a-1)x-ay=1\end{cases}$ has no solution, find the value of a.

10. 圖三所示為圓 $ABCDEFGH$，求 $a+b+c+d$ 的值。
Figure 3 shows the circle $ABCDEFGH$.
Find the value of $a+b+c+d$.



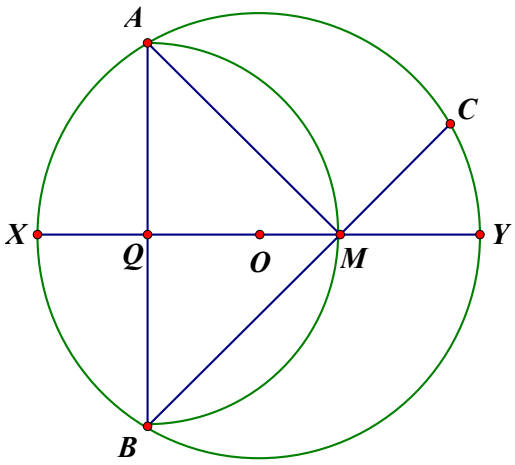
圖三 Figure 3

11. 若 $1+2+3+\cdots+k$ 的和為一完全平方 N，其中 $N<250\,000$，求 N 的最大可能值。
If $1+2+3+\cdots+k$ is a perfect square N, where $N<250\,000$,
find the largest possible value of N.

12. 若 ΔABC 的邊長為 9、10 及 17，求 ΔABC 外接圓的半徑。
If the lengths of the three sides of a ΔABC are 9, 10 and 17,
find the radius of the circum-circle of ΔABC.

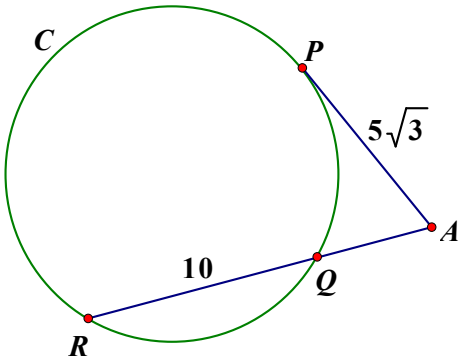
13. 求 $S=\dfrac{1}{2024}-\dfrac{3}{2024^2}+\dfrac{5}{2024^3}-\dfrac{7}{2024^4}+\dfrac{9}{2024^5}-\cdots$ 的值。
Find the value of $S=\dfrac{1}{2024}-\dfrac{3}{2024^2}+\dfrac{5}{2024^3}-\dfrac{7}{2024^4}+\dfrac{9}{2024^5}-\cdots$.

14. 在圖四中，XY 是一個以 O 為圓心及半徑為 5 cm 的圓的直徑。XY 與弦 AB 相交於點 Q，使得 $\angle AQO=90°$ 及 $XQ=QO$。以 AB 為直徑的半圓與 XY 相交於 M，延線 BM 與圓相交於點 C，求 AC 的長。
In Figure 4, XY is a diameter of the circle with centre at O and radius 5 cm. XY intersects the chord AB at Q such that $\angle AQO=90°$ and $XQ=QO$. A semi-circle with diameter AB intersects XY at M. BM produced intersects the circle at C. Find the length of AC.



圖四 Figure 4

15. 在圖五中，點 P 及 R 均在圓 C 上。AP 是 C 的切線及 AR 相交於 Q。若 $QR=10$ 及 $PA=5\sqrt{3}$，求 AQ 的長。
In Figure 5, P and R are points on the circle C. AP is the tangent to C at P and AR intersects at Q. If $QR = 10$ and $PA=5\sqrt{3}$, find the length of AQ.



圖五 Figure 5

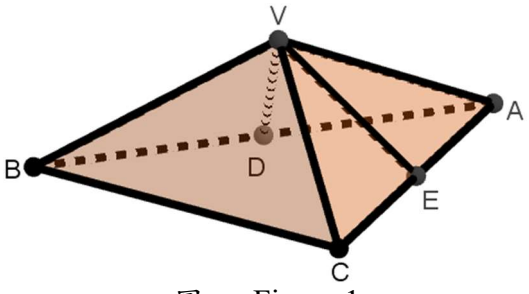
***** 試卷完 End of Paper *****

Hong Kong Mathematics Olympiad (2023-2024)
Heats – Group Event
香港數學競賽 (2023 – 2024)
初賽團體項目

除特別指明外，所有答案須以數字的真確值表達，並化至最簡。不接受近似值。所有附圖不一定依比例繪成。第一至五題，每題 2 分，第六至十題，每題 3 分。全卷滿分 25 分。
本卷由四位參賽同學共同作答。 時限：20 分鐘

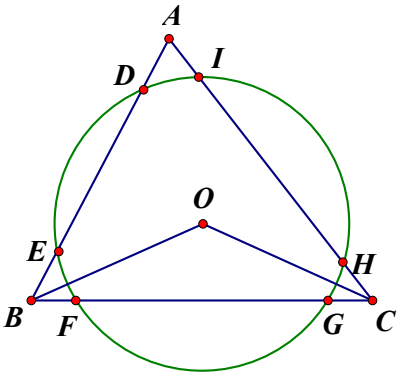
Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
No approximation is accepted. The diagrams are not necessarily drawn to scale.
Q1- Q5 2 marks each, Q6-Q10 3 marks each. The maximum mark for this paper is 25.
This paper is answered by 4 participants together. Time allowed: 20 minutes

1. 在圖一中，$VABC$ 是一個四面體，使得 $VA \perp VB$、$VB \perp VC$ 及 $VA \perp VC$。$VA = 5$，$VB = 4$ 及 $VC = 3$。若 D 和 E 分別為 AB 和 AC 的中點，求角錐 $VBCED$ 的體積。
In Figure 1, $VABC$ is a tetrahedron such that $VA \perp VB$, $VB \perp VC$ and $VA \perp VC$. $VA = 5$, $VB = 4$ and $VC = 3$. If D and E are the mid-points of AB and AC respectively, find the volume of pyramid $VBCED$.



圖一 Figure 1

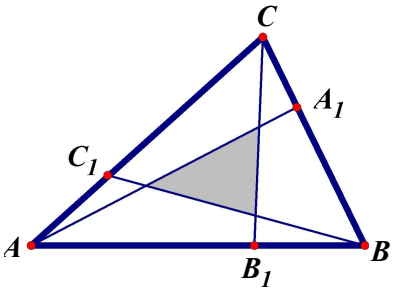
2. 在圖二中，O 是圓 $DEFGHI$ 的圓心，ΔABC 與該圓相交於 D、E、F、G、H 及 I，使得 $AD = EB$ 及 $BF = CG$，已知 $\angle ABO = 38°$ 及 $\angle ACO = 28°$，求 $\angle BOC$。
In Figure 2, O is the centre of the circle $DEFGHI$. ΔABC intersects the circle at D, E, F, G, H and I such that $AD = EB$ and $BF = CG$. If $\angle ABO = 38°$ and $\angle ACO = 28°$, find $\angle BOC$.



圖二 Figure 2

3. 設 a、b 及 c 為正整數。若 $ab + c = 2023$ 及 $a + bc = 2024$，求 $a + b + c$ 的值。
Let a, b and c be positive integers.
If $ab + c = 2023$ and $a + bc = 2024$, find the value of $a + b + c$.

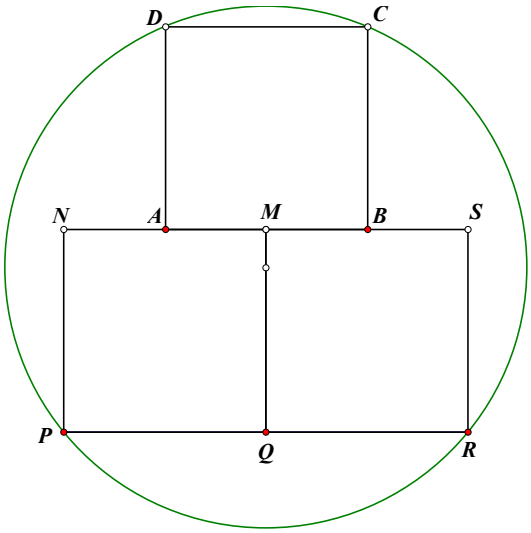
4. 在圖三中，A_1、B_1 及 C_1 分別為 BC、AC 及 AB 上的點，使得 $AC_1 = 2C_1B$，$BA_1 = 2A_1C$ 及 $CB_1 = 2B_1A$。若 ΔABC 的面積是 21 平方單位，求陰影部分的面積。
In Figure 3, A_1, B_1 and C_1 are points on BC, AC and AB respectively such that $AC_1 = 2C_1B$, $BA_1 = 2A_1C$ and $CB_1 = 2B_1A$. If the area of ΔABC is 21 square units, find the area of the shaded region.



圖三 Figure 3

5. 求方程 $\left(\log_4 x^2\right)^2 + 9\log_x 64 = \pi^{3\log_\pi 3}$ 的所有根之和。
Find the sum of the roots of the equation $\left(\log_4 x^2\right)^2 + 9\log_x 64 = \pi^{3\log_\pi 3}$.

6. 在圖四中，三個邊長為 1 cm 的正方形 *ABCD*、*MNPQ*、*MQRS* 併在一起使得 *A* 及 *B* 分別為 *MN* 及 *MS* 的中點。已知一圓包這三個正方形，且通過 *C*、*D*、*P* 及 *R*，求該圓的半徑。
In Figure 4, three squares *ABCD*, *MNPQ* and *MQRS* of sides1 cm toucheach other so that points *A* and *B* are the mid-points of *MN* and *MS* respectively. Given that a circle contains all three squares and passes through points *C*, *D*, *P* and *R*, find the radius of the circle.



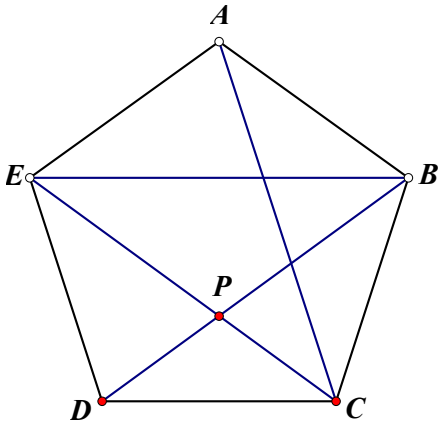
圖四 Figure 4

7. 若 *x*、*y* 及 *z* 為正整數，求能滿足 $xyz = 10000$ 的 (x, y, z)組的數目。
If *x*, *y* and *z* positive integers, find the number of sets of (x, y, z) satisfying $xyz = 10000$.

8. 設 *a* 為實數。若方程 $x^2 + ax + 6a = 0$ 有兩個整數解，求 *a* 的最大和最小值之差。
Let *a* be a real number. If the equation $x^2 + ax + 6a = 0$ has two integral roots, find the difference between the largest and the smallest values of *a* .

9. 若三角形的邊長為 25、39 及 56，求該三角形的內心及垂心之間的距離。
If the lengths of the three sides of a triangle are 25, 39 and 56, find the distance between the incentre and the orthocentre of the triangle.

10. 在圖五中，*ABCDE* 為一正五邊形，*BD* 及 *CE* 相交於 *P*。若ΔABE 的面積為 1，求ΔBPC 的面積。
In Figure 5, *ABCDE* is a regular pentagon, *BD* and *CE* intersect at *P*.If the area of ΔABE is 1, find the area of ΔBPC.



圖五 Figure 5

***** 試卷完 End of Paper *****

參加學校數目：212
初賽日期：2024 年 2 月 3 日星期六
試場一(HKI)：True Light Middle School of Hong Kong (43 隊)
試場二(KLN1)：Yan Chai Hospital Wong Wha San Secondary School (50 隊)
試場三(KLN2)：Newman Catholic College (41 隊)
試場四(NTE)： Shatin Government Secondary School (50 隊)
試場四(NTW)：Queen Elizabath School Old Students' Association Secondary School (28 隊)

Regional winners of the Heat Event

Hong Kong Island Region
HKI-10 Hong Kong Chinese Women's Club College 香港中國婦女會中學
HKI-17 Munsang College (Hong Kong Island) 港島民生書院
HKI-26 Singapore International School (Hong Kong) 新加坡國際學校(香港)
HKI-30 St Paul's Co-Educational College 聖保羅男女中學

Kowloon Region 1
KLN1-13 Fukien Secondary School 福建中學
KLN1-15 Good Hope School 德望學校
KLN1-19 Inno Secondary School 一諾中學
KLN1-25 La Salle College 喇沙書院
KLN1-41 Sing Yin Secondary School 聖言中學

Kowloon Region 2
KLN2-01 Buddhist Sin Tak College 佛教善德英文中學
KLN2-08 CNEC Christian College 中華傳道會安柱中學
NTE-11 Diocesan Boys' School 拔萃男書院
NTE-12 Diocesan Girls' School 拔萃女書院
NTE-28 Pui Ching Middle School 香港培正中學

New Territories East Region
NTE-02 Baptist Lui Ming Choi Secondary School 浸信會呂明才中學
NTE-15 HKTA Tang Hin Memorial Secondary School 香港道教聯合會鄧顯紀念中學
NTE-35 SKH Tsang Shiu Tim Secondary School 聖公會曾肇添中學
NTE-46 Wong Shiu Chi Secondary School 王肇枝中學

New Territories West Region
NTW-09 Po Leung Kuk Centenary Li Shiu Chung Memorial College 保良局百周年李兆忠紀念中學
NTW-17 Shun Tak Fraternal Association Yung Yau College 順德聯誼總會翁祐中學
NTW-22 Tung Wah Group of Hospital Lo Kon Ting Memorial College 東華三院盧幹庭紀念中學

School ID for Finals	School ID for Heats	Name of School
FE-01	NTE-02	Baptist Lui Ming Choi Secondary School
FE-02	NTE-03	Bishop Hall Jubilee School
FE-03	KLN2-01	Buddhist Sin Tak College
FE-04	NTE-08	Carmel Pak U Secondary School
FE-05	KLN2-08	CNEC Christian College
FE-06	HKI-06	CNEC Lau Wing Sang Secondary School
FE-07	NTE-11	Diocesan Boys' School
FE-08	NTE-12	Diocesan Girls' School
FE-09	KLN1-13	Fukien Secondary School
FE-10	KLN1-14	G.T. (Ellen Yeung) College
FE-11	KLN1-15	Good Hope School
FE-12	NTE-14	Heung To Middle School
FE-13	NTE-15	HKTA Tang Hin Memorial Secondary School
FE-14	KLN2-14	Ho Fung College (Sponsored by Sik Sik Yuen)
FE-15	KLN2-16	Hoi Ping Chamber of Commerce Secondary School
FE-16	NTE-16	HKBU Affiliated School Wong Kam Fai Secondary and Primary School
FE-17	HKI-10	Hong Kong Chinese Women's Club College
FE-18	KLN1-19	Inno Secondary School
FE-19	HKI-13	King's College
FE-20	KLN1-25	La Salle College
FE-21	HKI-17	Munsang College (Hong Kong Island)
FE-22	NTW-08	NTHYK Yuen Long District Secondary School
FE-23	KLN1-33	Po Leung Kuk No. 1 W.H. Cheung College
FE-24	NTW--09	Po Leung Kuk Centenary Li Shiu Chung Memorial College
FE-25	KLN1-37	Po Leung Kuk Laws Foundation College
FE-26	NTE-25	Po Leung Kuk Ma Kam Ming College
FE-27	NTW-11	Po Leung Kuk Tang Yuk Tien College
FE-28	NTE-28	Pui Ching Middle School
FE-29	NTE-29	Pui Kiu College
FE-30	HKI-19	Pui Kiu Middle School
FE-31	NTE-50	Queen Elizabeth School
FE-32	HKI-20	Queen's College
FE-33	NTE-35	SKH Tsang Shiu Tim Secondary School
FE-34	KLN2-31	Shun Tak Fraternal Association Lee Shau Kee College
FE-35	NTW-17	Shun Tak Fraternal Association Yung Yau College
FE-36	KLN1-41	Sing Yin Secondary School
FE-37	HKI-26	Singapore International School (Hong Kong)
FE-38	KLN2-32	SKH Lam Woo Memorial Secondary School
FE-39	HKI-30	St Paul's Co-Educational College
FE-40	HKI-31	St Paul's College
FE-41	KLN1-44	The Mission Covenant Church Holm Glad College
FE-42	KLN1-45	The Y.W.C.A. Hioe Tjo Yoeng College
FE-43	KLN2-34	True Light Girls' College
FE-44	KLN1-46	Tseung Kwan O Government Secondary School
FE-45	KLN2-35	Tsuen Wan Government Secondary School
FE-46	NTW-22	Tung Wah Group of Hospital Lo Kon Ting Memorial College
FE-47	KLN1-48	Tung Wah Group of Hospital Wong Fut Nam College
FE-48	KLN1-49	Wa Ying College
FE-49	NTE-46	Wong Shiu Chi Secondary School
FE-50	KLN2-40	Ying Wa College