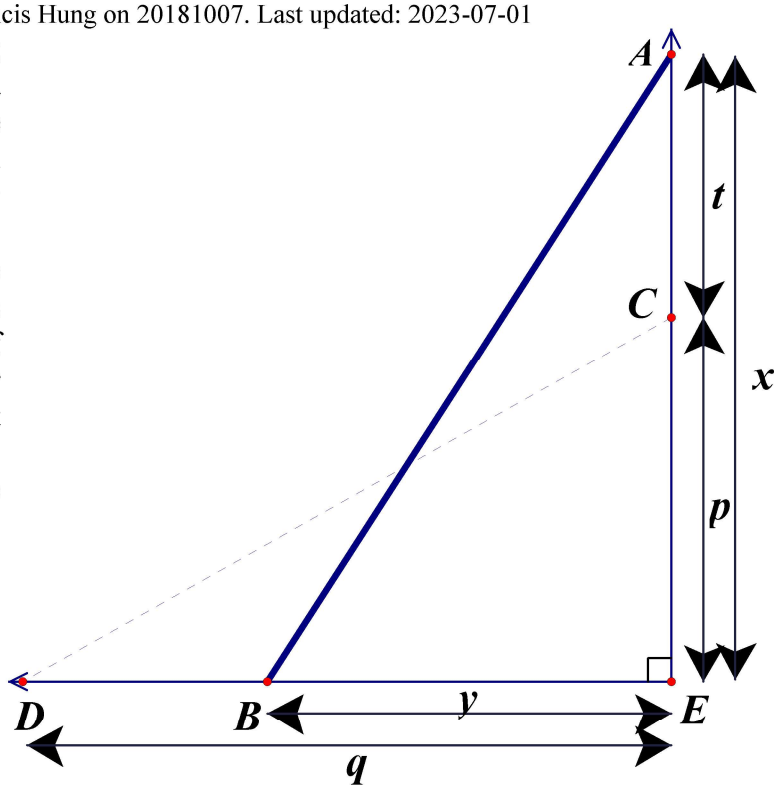
Fall of a ladder

Created by Mr. Francis Hung on 20181007. Last updated: 2023-07-01

In the figure, a ladder AB is leaning against a vertical wall, standing on a horizontal ground. The top of the ladder is x units above the ground and the foot of the ladder is y units away from the corner.
If the ladder slips down by t units so that the top of ladder is now p units above the ground and the foot of ladder is q units away from the corner E, find the distance moved of the foot of ladder BD in terms of x, y and t.
If x, y, p and q are integers, find the integral solutions.



$AB^2 = CD^2$
$x^2 + y^2 = p^2 + q^2$
$x^2 + y^2 = (x - t)^2 + q^2$
$q = \sqrt{x^2 + y^2 - (x - t)^2} = \sqrt{2xt - t^2 + y^2}$
$BD = q - y = \sqrt{2xt - t^2 + y^2} - y$

To find the integral solution, let $AB^2 = z^2$
If $a^2 + b^2 = c^2$ and $d^2 + e^2 = f^2$, let $z = cfk$, $x = afk$, $y = bfk$, $p = cdk$, $q = cek$
$t = x - p = afk - cdk = (af - cd)k$, $BD = q - y = cek - bfk = (ce - bf)k$, where k is a positive integer.

Example
If $z = 13 \times 5 = 65$, then $x = 12 \times 5 = 60$, $y = 5 \times 5 = 25$, $p = 3 \times 13 = 39$, $q = 4 \times 13 = 52$
$t = x - p = 60 - 39 = 21$, $BD = q - y = 52 - 25 = 27$