Example on variation by parts

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Reference:
http://www.td.gov.hk/en/road_safety/road_users_code/index/chapter_5_for_all_drivers/stopping_distance_/index.html

Suppose a driver wants to stop a car while he is driving.
The stopping distance (S m) is the sum of the thinking distance (x m) and the braking distance (y m)
The thinking distance varies as the speed (v km/h). The braking distance partly varies as v, partly varies as the square of v and partly varies as the cube of v.

Shortest stopping distance in metres

40 km/h Thinking distance + braking distance = stopping distance
vehicle speed 10m 10m 20m

60 km/h Thinking distance + braking distance = stopping distance
vehicle speed 15m 20m 35m

80 km/h Thinking distance + braking distance = stopping distance
vehicle speed 20m 40m 60m

(*a*) Express S in terms of x and y. 10 marks
(*b*) Express x in terms of v. 10 marks
(*c*) Express y in terms of v. 30 marks
(*d*) Show that $S = \frac{v^3}{9600} - \frac{v^2}{160} + kv$. Find k. 20 marks
(*e*) If the speed of the car is 100 km/h, find the stopping distance. 10 marks
(*f*) Suppose the car crashed in an accident. If the skid mark was 130 m, what was the least speed just before the crash? 20 marks

(*a*) $S = x + y$

(*b*) $x = \frac{1}{4}v$

(*c*) $y = av^3 + bv^2 + cv$

$$\begin{cases} a(40)^3 + b(40)^2 + c(40) = 10 \\ a(60)^3 + b(60)^2 + c(60) = 20 \\ a(80)^3 + b(80)^2 + c(80) = 40 \end{cases}$$

$$\begin{cases} 6400a + 160b + 4c = 1 \cdots\cdots(1) \\ 21600a + 360b + 6c = 2 \cdots\cdots(2) \\ 51200a + 640b + 8c = 4 \cdots\cdots(3) \end{cases}$$

(2) – (1), (3) – (2): $\begin{cases} 15200a + 200b + 2c = 1 \cdots\cdots(4) \\ 29600a + 280b + 2c = 2 \cdots\cdots(5) \end{cases}$

(5) – (4): $14400a + 80b = 1$ .....(6)

(3) – 2(1): $38400a + 320b = 2$ .....(7)

2(6) – (7)/2: $9600a = 1$

$a = \frac{1}{9600}$

Sub. $a = \frac{1}{9600}$ into (6): $\frac{14400}{9600} + 80b = 1$

$80b = -\frac{1}{2}$

$b = -\frac{1}{160}$

Sub. $a = \frac{1}{9600}$, $b = -\frac{1}{160}$ into (1)

$\frac{6400}{9600} - \frac{160}{160} + 4c = 1$

$4c = \frac{4}{3}$

$c = \frac{1}{3}$

$\therefore y = \frac{v^3}{9600} - \frac{v^2}{160} + \frac{v}{3}$

(*d*) $S = \frac{1}{4}v + \frac{v^3}{9600} - \frac{v^2}{160} + \frac{v}{3}$

$= \frac{v^3}{9600} - \frac{v^2}{160} + \frac{7v}{12}$

$k = \frac{7}{12}$

(*e*) When $v = 100$,

$S = \frac{1000000}{9600} - \frac{10000}{160} + \frac{700}{12}$

$S = 100$

The stopping distance is 100 m.

(*f*) Skid mark = 130 m $\rightarrow$ breaking distance = y m = 130 m

$$\frac{v^3}{9600} - \frac{v^2}{160} + \frac{v}{3} = 130$$

$v^3 - 60v^2 + 3200v - 1248000 = 0$

Let $f(v) = v^3 - 60v^2 + 3200v - 1248000$

$f(120) = 0$, $v - 120$ is a factor.

$f(v) = (v - 120)(v^2 + 60v + 10400) = 0$

$v = 120$

The minimum speed was 120 km/h