Given 1, 3, 5, 7, what is the next term?

Created by Mr. Francis Hung on 20210909 Last updated: 2021-09-09

The next term could be 9 or other numbers.

Let $f(n) = \frac{(n-2)(n-3)(n-4)}{(1-2)(1-3)(1-4)} \cdot 1 + \frac{(n-1)(n-3)(n-4)}{(2-1)(2-3)(2-4)} \cdot 3 + \frac{(n-1)(n-2)(n-4)}{(3-1)(3-2)(3-4)} \cdot 5 + \frac{(n-1)(n-2)(n-3)}{(4-1)(4-2)(4-3)} \cdot 7$

Then f(1) = 1, f(2) = 3, f(3) = 5, f(4) = 7 and

$f(5) = \frac{(5-2)(5-3)(5-4)}{(1-2)(1-3)(1-4)} \cdot 1 + \frac{(5-1)(5-3)(5-4)}{(2-1)(2-3)(2-4)} \cdot 3 + \frac{(5-1)(5-2)(5-4)}{(3-1)(3-2)(3-4)} \cdot 5 + \frac{(5-1)(5-2)(5-3)}{(4-1)(4-2)(4-3)} \cdot 7$

$= \frac{3 \cdot 2 \cdot 1}{(-1)(-2)(-3)} \cdot 1 + \frac{4 \cdot 2 \cdot 1}{(1)(-1)(-2)} \cdot 3 + \frac{4 \cdot 3 \cdot 1}{(2)(1)(-1)} \cdot 5 + \frac{4 \cdot 3 \cdot 2}{(3)(2)(1)} \cdot 7$

$= -1 + 12 - 30 + 28 = 11 - 2 = 9$

Is it a coincidence?

Let $g(n) = \frac{(n-2)(n-3)(n-4)(n-5)}{(1-2)(1-3)(1-4)(1-5)} \cdot 1 + \frac{(n-1)(n-3)(n-4)(n-5)}{(2-1)(2-3)(2-4)(2-5)} \cdot 3 + \frac{(n-1)(n-2)(n-4)(n-5)}{(3-1)(3-2)(3-4)(3-5)} \cdot 5$
$+ \frac{(n-1)(n-2)(n-3)(n-5)}{(4-1)(4-2)(4-3)(4-5)} \cdot 7 + \frac{(n-1)(n-2)(n-3)(n-4)}{(5-1)(5-2)(5-3)(5-4)} \cdot 20$

Then g(1) = 1, g(2) = 3, g(3) = 5, g(4) = 7, but g(5) = 20

Exercise: Find the general term of a sequence such that $a_1 = 1$, $a_2 = 3$, $a_3 = 5$, $a_4 = 7$, but $a_5 = 19$.