Supplementary Exercise on A.P and G.P.

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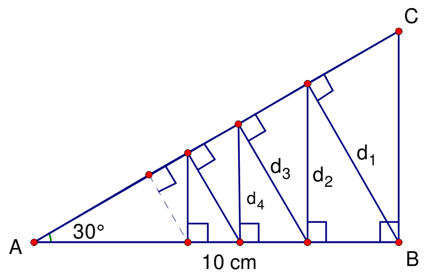
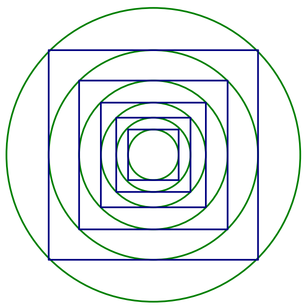
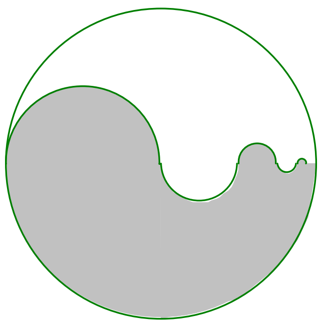
1. Between x and y there are 4 arithmetic means and also 3 arithmetic means. The sum of the four exceeds the sum of the three by 10, and the first of the three exceeds the first of the four by $\frac{1}{2}$. Find x and y .
2. Four numbers are in A.P.. The product of the second and third exceeds the product of the other two by 32, and the product of the second and the fourth exceeds the product of the other two by 72. Find the numbers.
3. After the brake applied, a car travelled 100 m in the first second. In each later second, the car travelled $\frac{1}{4}$ of the distance travelled in the previous second.
   (a) Find the distance the car travelled during the 2nd second.
   (b) Find the car travelled during the nth second.
   (c) Find the total distance the car travelled at the end of the nth second.
   (d) Find the total distance travelled from the moment when the brake was applied to the moment when it come to rest.
4. Between 1 and 101 , find the sum of
   (a) all the even numbers;
   (b) all the numbers which are divisible by 14 ;
   (c) all the even numbers which are not divisible by 7 .
5. In the figure, ΔABC is right-angled at B, $\angle A = 30°$, and $AB = 10$ cm . d_1 , d_2 , d_3 , $\cdots$ are the lengths of the perpendiculars to AB and AC .
   (a) Find d_1 and d_2.
   (b) Find $d_1 + d_2 + d_3 + \cdots$ if the perpendiculars are drawn indefinitely.



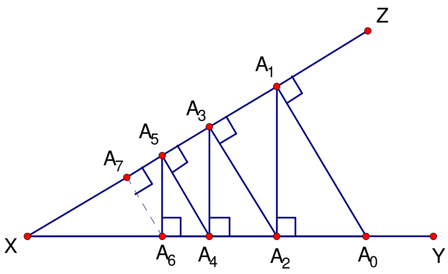
6. In the figure, each square is inscribed in a circle and circumscribes another circle. The process is continued infinitely. If the radius of the largest circle is 20 cm, find the sum of
   (a) the circumference of the circles,
   (b) the areas of the squares.



7. In the figure, the circle is divided by the semi-circular arcs whose radii are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and so on of that of the circle. Find the ratio of the areas of the unshaded part to the shaded part.



8. In the figure, XY and XZ are two straight lines cut at X. From any point A_0 on XY, draw a ⊥ to cut XZ at A_1, and from A_1 draw a ⊥ to cut XY at A_2, and from A_2 draw a ⊥ to cut XZ at A_3, and so on. If $A_0A_1 = 7$ cm, $A_1A_2 = 6$ cm, find the total lengths of the ⊥s, if the process continues indefinitely.



9. A person saves each year \$200 more than he saves in the preceding year, and he saves \$400 in the first year. At least how many years would it take for his savings, not including interest, to amount more than \$200000?

10. A boy arranges rows of marbles one against the other so that each row contains one marble less than the preceding. The last row consists of one marble only, which forms the apex of a triangle. If the boy has 153 marbles, how many marbles are there in the last row of the biggest triangle he can construct?

11. (a) Mary works in a company. In the first year, her monthly salary is \$550. After each year of completed service, her monthly salary is increased by \$35. If she works in the company for 10 years, find the total amount of salary she receives.

(b) A student was asked to sum to infinity the following geometric progression 9, 3, 1, ⋯. He found that the sum of the first 6 terms only and gave that sum as the required answer. Find (i) his error,

(ii) his percentage error, correct to two significant figures. (1976)

12. In the first second after the engine was shut off, a train travelled 20 m. In every successive second, the train travelled 98% of the distance covered in the previous second. (1977)

(a) Find the distance the train travelled in the n^{th} second.

(b) Find the total distance the train travelled in the first n seconds.

(c) Hence, or otherwise, find the total distance the train travelled until it came to a stop.

13. In an arithmetic series, the sum of the first 3 terms is 12, and their product is 28. Find the first term and the common difference of this series. (1977)

14. Three numbers x, $(x-1)$ and $(2x-1)$ form an arithmetic progression. Calculate the value of x. (1978)

15. Four numbers a, b, x, c form an arithmetic progression.

(a) Express x in terms of a and b.

(b) Express x in terms of a and c. (1978)

16. x, y, a, b, z form an arithmetic progression. Find

(a) a in terms of x and y.

(b) b in terms of x and z.

17. $(a-3)$, $(2a-5)$ and $(5a+3)$ form an arithmetic progression. Find the first term.

18. Tom, Dick and Henry are to share a square piece of chocolate in the following way:
In the first operation, they divide the piece of chocolate into 4 equal square portions. Each person gets a portion and one portion is left over (See Figure 5).
In the second operation, they divide the remaining portion again into 4 equal square portions. Each person gets a portion and again one portion is left over (See Figure 6).
They perform the third, fourth, fifth, $\cdots$, nth operation in the same way.

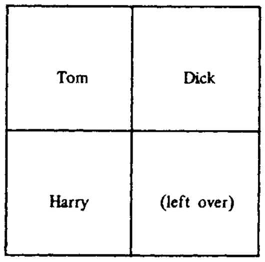


Figure 5

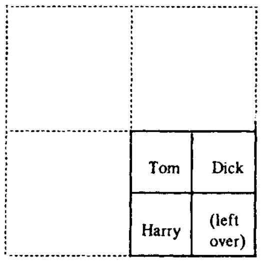
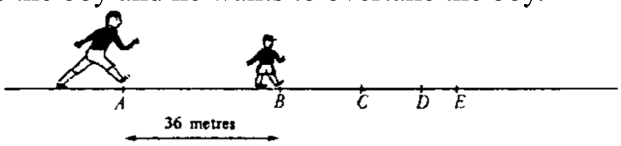


Figure 6

(a) What fraction of the original piece of chocolate does Tom get
  (i) in the first operation,
  (ii) in the second operation alone,
  (iii) in the nth operation alone?
(b) If it is possible to perform an infinite number of operations, show, by summing a series, that Tom will totally get $\frac{1}{3}$ of the original piece of chocolate. (1978)

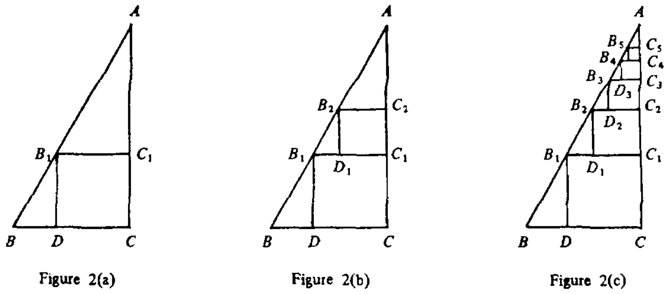
19. On a straight road, a man is 36 metres behind a boy. They run at uniform speeds, the man runs three times as fast as the boy and he wants to overtake the boy.



(a) In order to find the distance the man must run before overtaking the boy, the method of progression may be used as follows:
At the start, the mean is at A and the boy is at B. (See the figure)
When the man comes to B, the boy will be at C.
When the man comes to C, the boy will be at D, and so on.
The distances AB, BC, CD, DE, $\cdots$ between the man and the boy will be come shorter and shorter.
In fact, the distances form a geometric progression and the required distance is the sum of the progression to infinity.
  (i) Find the common ratio of this geometric progression.
  (ii) Write down the first three terms of the progression.
  (iii) Find the required distance.
(b) Find the distance the man must run before overtaking the boy without using the method of progression. (1979)

20. Let $k > 0$.
(a) (i) Find the common ratio of the geometric progression k, $10k$, $100k$, $\cdots$.
  (ii) Find the sum of the first terms of the geometric progression k, $10k$, $100k$, $\cdots$.
(b) (i) Show that $\log_{10} k$, $\log_{10} 10k$, $\log_{10} 100k$ is an arithmetic progression.
  (ii) Find the sum of the first n terms of the arithmetic progression $\log_{10} k$, $\log_{10} 10k$, $\log_{10} 100k$, $\cdots$. Also, if $n = 10$, what is the sum? (1980)

21.



In the figure 2(a), B_1C_1CD is a square inscribed in the right-angled triangle ABC, $\angle C = 90°$, $BC = a$, $AC = 2a$, $B_1C_1 = b$.

(a) Express b in terms of a.

(b) $B_2C_2C_1D_1$ is a square inscribed in ΔAB_1C_1 (see Figure 2(b)).

 (i) Express B_2C_2 in terms of b.

 (ii) Hence express B_2C_2 in terms of a.

(c) If squares $B_3C_3C_2D_2$, $B_4C_4C_3D_3$, $B_5C_5C_4D_4$, $\cdots$ are drawn successively as indicated in Figure 2(c).

 (i) Write down the length of B_5C_5 in terms of a .

 (ii) Find, in terms of a, the sum of the areas of the infinitely many squares drawn in this way. (1981)

22. (a) (i) Find the sum of all the multiples of 3 from 1 to 1000.

 (ii) Find the sum of all the multiples of 4 from 1 to 1000 (including 1000).

(b) Hence, or otherwise, find the sum of all the integers from 1 to 1000 (including 1 and 1000) which are neither multiples of 3 nor multiples of 4. (1982)

23. I propose to take 30 consecutive terms of the series $100+99+98+97+\cdots$. At which term must I begin that their sum may be 1155?

24. If P, Q, R are the pth, qth, and rth terms of an Arithmetic sequence,

 prove that $p\ (Q - R) + q\ (R - P) + r\ (P - Q) = 0$

25 In an Arithmetic sequence, the first term is 1, the number of terms is odd. The sum of all terms with odd numbers is 176, and the sum of all terms with even numbers is 154. Find the series.

26. In an infinite geometric series in which all the terms are positive, show that the sum cannot be less than four times the second term of the series.

27. A man climbing a mountain ascends 400 m in the first hour. In the successive hours, he ascends $\frac{3}{4}$ of the distance which he ascends in the previous hour. If he reaches the top in 4 hours, find the height of the mountain.

28. Find the total distance travelled for an elastic ball from start to rest if it is dropped from a height of 20 m, and after each fall it rebounds to $\frac{3}{4}$ of the height from which it falls.

29. The output of a silver mine in a year is equal to 70% of the output of the previous year. Find the total possible output if the output of the first year weighs 12 tons.

1. Suppose the 4 means are $x + h$, $x + 2h$, $x + 3h$, $x + 4h$; and the 3 means are $x + k$, $x + 2k$, $x + 3k$.
Then their respective sum are
$4x + 10h$ and $3x + 6k$.
$\therefore 4x + 10h = 3x + 6k + 10$
or $x + 10h - 6k = 10 \cdots\cdots (1)$
and $x + k = x + h + \frac{1}{2}$
or $2(k - h) = 1 \cdots\cdots (2)$
Now $x + 5h = y$
and $x + 4k = y$
$\therefore h = \frac{y-x}{5}$
and $k = \frac{y-x}{4}$
putting these in (1),
Simplifying we have
$x + y = 20 \cdots\cdots (3)$
$-x + y = 10 \cdots\cdots (4)$
Solving (3) and (4)
$\therefore x = 5, y = 15$

2. Let the four numbers be a, $a + d$, $a + 2d$, $a + 3d$, then
$$\begin{cases}(a+d)(a+2d)=a(a+3d)+32\\(a+d)(a+3d)=a(a+2d)+72\end{cases}$$
$$\begin{cases}2d^2=32\\2ad+3d^2=72\end{cases}$$
$d = \pm 4$
When $d = 4$, $a = 3$, when $d = -4$, $a = -3$
The numbers are 3, 7, 11, 15 or –3, –7, –11, –15 .

3. (a) $100 \text{ m} \times \frac{1}{4} = 25 \text{ m}$

(b) $100 \times \left(\frac{1}{4}\right)^{n-1}$ m

(c) $100 + 100 \times \frac{1}{4} + \cdots + 100 \times \left(\frac{1}{4}\right)^{n-1} = \frac{100\left[1-\left(\frac{1}{4}\right)^n\right]}{1-\frac{1}{4}}$, $a = 100$, $r = \frac{1}{4}$
$$= \frac{400\left[1-\left(\frac{1}{4}\right)^n\right]}{3} \text{ m}$$

(d) sum to infinity $= 100 + 100 \times \frac{1}{4} + \cdots$ to infinity, $a = 100$, $r = \frac{1}{4}$
$$= \frac{100}{1-\frac{1}{4}} = \frac{400}{3} = 133\frac{1}{3} \text{ m}$$

4. (a) $S_{50} = 2 + 4 + 6 + \cdots + 100 = \frac{1}{2} \cdot 50(2 + 100) = 2550$

(b) $S_7 = 14 + 28 + \cdots + 98 = \frac{1}{2} \cdot 7(14 + 98) = 392$

(c) Sum of all the even numbers which are not divisible by 7
$= S_{50} - S_7 = 2550 - 392 = 2158$

5. (a) $BC = 10 \tan 30° = \frac{10}{\sqrt{3}}$ cm

$d_1 = BC \cos 30° = 5$ cm

$d_2 = d_1 \cos 30° = \frac{5\sqrt{3}}{2}$ cm

(b) $d_1, d_2, d_3, d_4, \ldots$ from a G.P. with the first term = 5 cm, $r = \frac{\sqrt{3}}{2}$ cm $(0 < r < 1)$

$s_\infty = d_1 + d_2 + d_3 + \cdots$ to infinity $= \frac{a}{1-r} = \frac{5}{1-\frac{\sqrt{3}}{2}}$

$= \frac{10}{2-\sqrt{3}} = \frac{10}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = 10\left(2+\sqrt{3}\right)$cm

6. (a) The radii of the circles are 20, 20 sin 45°, 20 sin^2 45°, …

Sum of the circumferences

$= 2\pi\left[20 + 20 \cdot \frac{1}{\sqrt{2}} + 20\left(\frac{1}{\sqrt{2}}\right)^2 + \cdots\right]$ cm

$= 2\pi \cdot \frac{20}{1-\frac{1}{\sqrt{2}}}$ cm

$= 40\pi\left(2+\sqrt{2}\right)$cm

(b) The lengths of the sides of squares are 2(20 sin 45°), 2(20 sin^2 45°), 2(20 sin^3 45°)⋯

Sum of the areas

$= 1600\left[\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^6 + \cdots\right]$ cm^2

$= 1600\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right)$ cm^2

$= 1600 \cdot \frac{\frac{1}{2}}{1-\frac{1}{2}}$ cm^2

= 1600 cm^2

7. The area of the shaded part $= \frac{1}{2}\pi R^2 + \frac{1}{2}\pi\left(\frac{R}{2}\right)^2 - \frac{1}{2}\pi\left(\frac{R}{4}\right)^2 + \frac{1}{2}\pi\left(\frac{R}{8}\right)^2 - \cdots$

$= \frac{1}{2}\pi R^2\left(1 + \frac{1}{4} - \frac{1}{16} + \frac{1}{64} - \cdots\right)$

$= \frac{1}{2}\pi R^2\left[1 + \frac{\frac{1}{4}}{1-\left(-\frac{1}{4}\right)}\right] = \frac{3}{5}\pi R^2$

∴ The ratio of areas is 2 : 3 .

8. $\Delta A_0A_1A_2 \sim \Delta A_1A_2A_3 \sim \Delta A_2A_3A_4 \sim \cdots$ and $A_0A_1 = 7$ cm, $A_1A_2 = 6$ cm

∴ $\frac{A_1A_2}{A_0A_1} = \frac{A_2A_3}{A_1A_2} = \frac{A_3A_4}{A_2A_3} = \cdots = \frac{6}{7}$

∴ $A_1A_2 = 7 \times \frac{6}{7}$ cm.

$A_2A_3 = A_1A_2 \times \frac{6}{7} = 7 \cdot \left(\frac{6}{7}\right)^2$ cm

$A_3A_4 = A_2A_3 \times \frac{6}{7} = 7 \cdot \left(\frac{6}{7}\right)^3$ cm

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The total length = $[7 + 7 \cdot \left(\frac{6}{7}\right) + 7 \cdot \left(\frac{6}{7}\right)^2 + \cdots]$cm $= \frac{7}{1-\frac{6}{7}}$ cm = 49 cm

9. Let n be the required number of years

$a = 400$, $d = 200$, $S_n = 200000$

$\therefore \ \frac{n}{2}[2\times 400+(n-1)\times 200]=200000$

$n(400 + 100n - 100) = 200000$

$n(n + 3) = 2000$

$n^2 + 3n - 2000 = 0$

$n=\frac{-3\pm\sqrt{8009}}{2}=\frac{-3\pm 89.5}{2}$ = 43.25 or –46.25 (rejected)

It would take at least 44 years to amount more than \$200000 for his saving.

10. Suppose there are n rows, then the total number of marbles will be:

$1 + 2 + 3 + \cdots$ to n terms $=\frac{n(n+1)}{2}=153$

$\therefore\ n(n + 1) = 306$

$n^2 + n - 306 = 0$

$(n - 17)(n + 18) = 0$

$n = 17$ or -18 (rejected)

$\therefore$ The number of marbles in the last row is 17.

11. (a) 1st year's salary = \$550×12 = \$6600

2nd year's salary = \$(550 + 35)×12 = \$7020

3rd year's salary = \$(550 + 35 + 35)×12 = \$7440

The amount of money form an Arithmetic sequence with a = \$6600, d = \$420

$\therefore$ The total salary for 10 years = \$$\frac{10}{2}[2\times 6600+(10-1)\times 420]$ = \$84900

(b) 9, 3, 1, $\cdots$ forms a geometric sequence with $a = 9$, $r=\frac{1}{3}$.

(i) $S_6=\frac{9\left[1-\frac{1}{3^6}\right]}{1-\frac{1}{3}}=\frac{27\left[1-\frac{1}{729}\right]}{2}$

$=\frac{364}{27}=13\frac{13}{27}$

$S_\infty=\frac{9}{1-\frac{1}{3}}=\frac{27}{2}$ (= 13.5)

His error $=\frac{27}{2}-13\frac{13}{27}=\frac{1}{54}$

(ii) Percentage error $=\frac{\frac{1}{54}}{\frac{27}{2}}\times 100\% = 0.14\%$

12. (a) Distance travelled by the train in successive seconds are 20 m, 20(0.98) m, $20(0.98)^2$ m, $\cdots$. They form a Geometric sequence with a = 20 m r = 0.98

$\therefore$ Distance travelled in the n^{th} second = $20\times 0.98^{n-1}$ m

(b) Total distance travelled in the first n seconds

$S_n=\frac{20\left[1-0.98^n\right]}{1-0.98}$ m = $1000(1 - 0.98^n)$ m

(c) Total distance travelled until it stopped:

$S_\infty=\frac{20}{1-0.98}$ m = 1000 m

13. Let the three terms be $a - d, a, a + d$.
$a - d + a + a + d = 12 \Rightarrow a = 4$
$(a - d)\, a\, (a + d) = 28$
$4(16 - d^2) = 28$
$16 - d^2 = 7$
$d^2 = 9$
$d = \pm 3$
The first term is 1, common difference is 3 or the first term is 7 and the common difference is -3 .

14. $2(x - 1) = x + 2x - 1$
$x = -1$

15. (a) $x - b = b - a$
$x = 2b - a$
(b) Let d = common difference
$c = a + 3d \Rightarrow d = \frac{c-a}{3}$
$x = a + 2d = a + 2\left(\frac{c-a}{3}\right)$
$= \frac{a+2c}{3}$

16. (a) $a - y = y - x$
$a = 2y - x$
(b) Let d = common difference
$z = x + 4d \Rightarrow d = \frac{z-x}{4}$
$b = x + 3d = x + 3\left(\frac{z-x}{4}\right)$
$= \frac{x+3z}{4}$

17. $2(2a - 5) = (a - 3) + (5a + 3)$
$-10 = 2a$
$a = -5$
The first term is $a - 3 = -8$

18. (a) (i) $\frac{1}{4}$ of the original piece.
(ii) $\frac{1}{4^2} = \frac{1}{16}$ of the original piece.
(iii) $\frac{1}{4^n}$ of the original piece.
(b) The sum the Geometric series $= \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^n} + \cdots$ to infinity
$= \frac{\frac{1}{4}}{1 - \frac{1}{4}}$
$= \frac{1}{4} \cdot \frac{4}{3}$
$= \frac{1}{3}$

19. (a) (i) Let x be the speed of the boy, then $3x$ is the speed of the man. Suppose t is the time required for the man and the boy to reach B and C respectively.

$AB = 3xt$

$BC = xt$

$\therefore$ The common ratio of the G.P. $= \dfrac{BC}{AB}$

$= \dfrac{xt}{3xt}$

$= \dfrac{1}{3}$

(ii) $T_1 = 36$ m

$T_2 = \dfrac{1}{3} \times 36 = 12$ m

$T_3 = \dfrac{1}{3} \times 12 = 4$ m

(iii) The required distance:

$S_\infty = 36(\dfrac{1}{1-\frac{1}{3}})$

$= 54$ m

(b) Let d be the required distance.

Time taken by the man $= \dfrac{d}{3x}$

Time taken by the boy $= \dfrac{d-36}{x}$

$\therefore \dfrac{d}{3x} = \dfrac{d-36}{x}$

$\dfrac{d}{3} = d - 36$

$d = 54$ m

20. (a) (i) Let r be the common ratio of the G.P. k, $10k$, $100k$,

$\because k > 0 \Rightarrow r = \dfrac{10k}{k}$

$= 10$

(ii) Let S be sum of the first n terms of the G.P. k, $10k$, $100k$, …

$S = \dfrac{a(r^n - 1)}{r-1}$

$a = k$, $r = 10$

$S = \dfrac{k(10^n - 1)}{10-1}$

$= \dfrac{k(10^n - 1)}{9}$

(b) (i) $\log_{10}100k - \log_{10}10k = \log_{10}\dfrac{100k}{10k}$

$= \log_{10}10$

$= 1$

$\log_{10}10k - \log_{10}k = \log_{10}\dfrac{10k}{k}$

$= \log_{10}10$

$= 1$

$\therefore$ $\log_{10}k$, $\log_{10}10k$, $\log_{10}100k$ is an arithmetic progression.

(ii) Let S be the sum of the first n terms of an A.P. $\log_{10} k, \log_{10} 10k, \log_{10} 100k \ldots$

$S = \frac{n}{2}[2a + (n-1)d]$

$a = \log_{10} k$, $d = \log_{10} 10 = 1$

$S = \frac{n}{2}[2\log_{10} k + (n-1)1]$

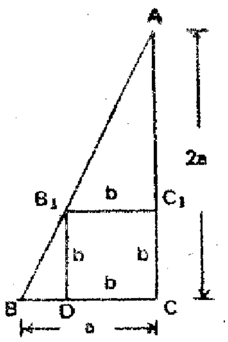
If $n = 10$

$S = \frac{10}{2}[2\log_{10} k + (10-1)]$

$= \frac{10}{2}[2\log_{10} k + 9]$

$= 10\log_{10} k + 45$

21. (a)



$\Delta AB_1C_1 \sim \Delta ABC$

$\therefore \frac{AC_1}{B_1C_1} = \frac{AC}{BC}$

$\frac{2a-b}{b} = \frac{2a}{a}$

$\therefore 2a - b = 2b$

$\therefore 3b = 2a$

$b = \frac{2}{3}a$

(b) (i) $B_2C_2 = \frac{2}{3}B_1C_1$

$= \frac{2}{3}b$

(ii) $B_2C_2 = \frac{2}{3}\left(\frac{2}{3}a\right)$

$= \frac{4}{9}a$

(c) (i) $\because B_1C_1 = \frac{2}{3}a$

$B_2C_2 = \frac{4}{9}a$

$B_3C_3 = \frac{8}{27}a$

$\therefore$ they are in G.P. with $T_1 = \frac{2}{3}a$

$r = \frac{2}{3}$

$\therefore B_5C_5 = T_1r^4$

$= \frac{2}{3}a\left(\frac{2}{3}\right)^4$

$= \frac{32}{243}a$

(ii) Area of square $B_1C_1CD = \left(\frac{2}{3}a\right)^2$

Area of square $B_2C_2C_1D_1 = \left(\frac{4}{9}a\right)^2$

Area of square $B_3C_3C_2D_2 = \left(\frac{8}{27}a\right)^2$

$\therefore$ They are in G.P. with

$T_1 = \left(\frac{2}{3}a\right)^2$

$r = \left(\frac{4}{9}a\right)^2 \div \left(\frac{2}{3}a\right)^2$

$= \frac{4}{9}$

$\therefore S_\infty = \frac{T_1}{1-r}$

$= \frac{\left(\frac{2}{3}a\right)^2}{1-\frac{4}{9}}$

$= \frac{4}{5}a^2$

22. (a) (i) Let S_1 = the sum of all the multiples of 3 from 1 to 1000 .
Let $a = 3$, $d = 3$
$T_n = 999$
$\therefore 999 = 3 + (n - 1)(3)$
$n = 333$
$\therefore$ There are 333 numbers which are multiples of 3 between 1 and 1000.
$\therefore S_1 = \frac{333}{2}[2(3) + (333 - 1)3]$
$= 166833$

(ii) Let S_2 = the sum of all the multiples of 12 between 1 and 1000.
Let $a = 4$, $d = 4$
$T_n = 1000$
$\therefore 1000 = 4 + (n - 1)(4)$
$n = 250$
$\therefore$ There are 250 numbers which are multiples of 4 between 1 and 1000.
$\therefore S_2 = \frac{250}{2}[2(4) + (250 - 1)4]$
$= 125500$

(b) Let S_3 = the sum of all the multiples of 12 from 1 to 1000.
The greatest number which is less than 1000 and divisible by 12 is 996.
Let $a = 12$, $d = 12$
$T_n = 996$
$\therefore$ $996 = 12 + (n - 1)(12)$
$n = 83$
$\therefore$ There are 83 numbers which are multiples of 12 between 1 and 1000.
$\therefore$ $S_3 = \frac{83}{2}[2(12) + (83 - 1)(12)]$
$= 41832$
Let S = the sum of all integers from 1 to 1000.
$\therefore$ $S = \frac{1000}{2}[2(1) + (1000 - 1)(1)]$
$= 500500$
The required sum $= S - (S_1 + S_2 - S_3)$
$= 500500 - (166833 + 125500 - 41832)$
$= 249999$

23. Let a be the number I begin at, then
$\frac{30}{2}[2a + 29(-1)] = 1155$
$a = 53$
Thus I must begin at 53, which is the 48th term.

24. Let a be the first term. d the common difference.
$P = a + (p - 1)d$ ············ (1)
$Q = a + (q - 1)d$ ············ (2)
$R = a + (r - 1)d$ ············ (3)
From (1) and (2)
$P - Q = (p - q)d$
From (2) and (3)
$Q - R = (p - r)d$
From (3) and (1)
$R - P = (r - p)d$
$\therefore$ $p(Q - R) + q(R - P) + r(P - Q)$
$= p(p - r)d + q(r - p)d + r(p - q)d$
$= 0$

25. Suppose there are $2n + 1$ terms.
no. of terms with odd nos. $= n + 1$
no. of terms with even nos. $= n$
For all terms with odd numbers:
$a = 1$
C.D. $= 2d$
$\therefore$ $\frac{n+1}{2}[2\times1 + (\overline{n+1} - 1)2d] = 176$
$\therefore$ $(n + 1)(1 + nd) = 176$ ········· (1)
For all terms with even numbers
$a = 1 + d$
C.D. $= 2d$
$\therefore$ $\frac{n}{2}[2(1 + d) + (n - 1)2d] = 154$
$\therefore$ $n(1 + nd) = 154$ ········· (2)

From (1)

$n(1 + nd) + (1 + nd) = 176$

putting (2) in it

$nd = 21$ ……… (3)

From (1)

$(n + 1)(1 + 21) = 176$

$\therefore n = 7$

$\therefore d = \frac{21}{n} = \frac{21}{7} = 3$

26. Let $a + ar + ar^2 + \cdots$ be the series. Then we have to show that $\frac{a}{1-r}$ is not less than $4ar$.

i.e. to show that $\frac{a}{1-r} - 4ar$ is positive.

Now $\frac{a}{1-r} - 4ar = \frac{a(1-2r)^2}{1-r}$

a is positive and $(1 - 2r)^2$ is positive. Also $1 - r$ is positive, since $r < 1$.

$\therefore$ $\frac{a(1-2r)^2}{1-r}$ is positive which proves $\frac{a}{1-r}$ is not less than $4ar$.

27. The height of the mountain $= \dfrac{400[1-(\frac{3}{4})^4]}{1-\frac{3}{4}}$

$= \dfrac{400(\frac{4^4-3^4}{4^4})}{\frac{1}{4}}$

$= \dfrac{25(256-81)}{4}$

$= \dfrac{25(175)}{4}$

$= 1093.75$ (m)

28. The total distance travelled

$= 20 + 2\cdot[20\left(\frac{3}{4}\right) + 20\left(\frac{3}{4}\right)^2 + ...]$

$= 20 + 2\cdot \dfrac{20\left(\frac{3}{4}\right)}{1-\frac{3}{4}}$

$= 20 + 2\cdot \dfrac{20\left(\frac{3}{4}\right)}{\frac{1}{4}}$

$= 20 + 120$

$= 140$ m

29. The total possible output

$= 12 + 12(0.7) + 12(0.7)^2 + \cdots$

$= \dfrac{12}{1-0.7}$

$= \dfrac{12}{0.3}$

$= 40$ tons