Formulae for the Trigonometric functions

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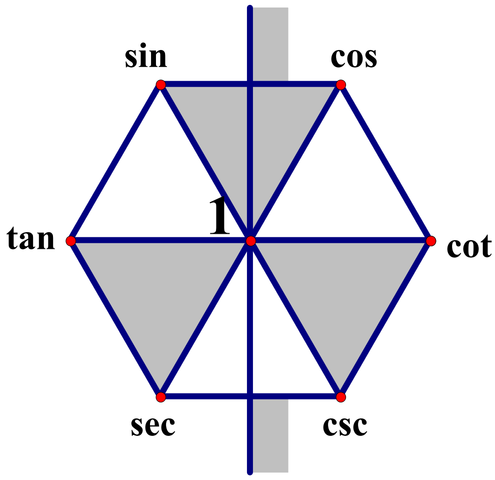
I The magic hexagon:

Along each diagonal,

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sin\theta = \dfrac{1}{\csc\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$



The S family The C family

In each shaded triangle,

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

In any three adjacent vertices,

$\sin\theta = \cos\theta \cdot \tan\theta$ $\cos\theta = \sin\theta \cdot \cot\theta$

$\cot\theta = \cos\theta \cdot \csc\theta$ $\csc\theta = \cot\theta \cdot \sec\theta$

$\sec\theta = \tan\theta \cdot \csc\theta$ $\tan\theta = \sin\theta \cdot \sec\theta$

II General Solutions

$\sin\theta = \sin\alpha$, $\theta = 180°n + (-1)^n\alpha$ $\theta = n\pi + (-1)^n\alpha$, where n is an integer.

$\cos\theta = \cos\alpha$, $\theta = 360°n \pm \alpha$ $\theta = 2n\pi \pm \alpha$, where n is an integer.

$\tan\theta = \tan\alpha$, $\theta = 180°n + \alpha$ $\theta = n\pi + \alpha$, where n is an integer.

III Compound Angle Formulae

$\sin(A + B) = \sin A\cos B + \cos A\sin B$ SC + CS

$\sin(A - B) = \sin A\cos B - \cos A\sin B$ SC – CS

$\cos(A + B) = \cos A\cos B - \sin A\sin B$ CC – SS

$\cos(A - B) = \cos A\cos B + \sin A\sin B$ CC + SS

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$$\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan A\tan C}$$

IV Multiple angles

$\sin 2\theta = 2\sin\theta\cdot\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

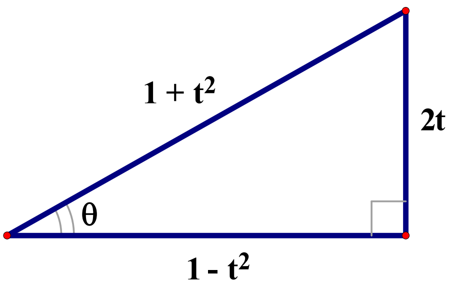
$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$

V Half angles

Let $t=\tan\frac{\theta}{2}$, then $\sin\theta=\frac{2t}{1+t^2}$

$$\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\tan\theta=\frac{2t}{1-t^2}$$



$$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$$

VI Sum and Product

Sum

$$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Product

$$\sin X\cos Y=\frac{1}{2}\left[\sin(X+Y)+\sin(X-Y)\right]$$

$$\cos X\sin Y=\frac{1}{2}\left[\sin(X+Y)-\sin(X-Y)\right]$$

$$\cos X\cos Y=\frac{1}{2}\left[\cos(X+Y)+\cos(X-Y)\right]$$

$$\sin X\sin Y=-\frac{1}{2}\left[\cos(X+Y)-\cos(X-Y)\right]$$

VII Differentiation: In each shaded triangle's edge,

$$\text{DS}=+\quad\frac{\mathrm{d}\sin x}{\mathrm{d}x}=\cos x \qquad\qquad \text{DC}=-\quad\frac{\mathrm{d}\cos x}{\mathrm{d}x}=-\sin x$$

$$\frac{\mathrm{d}\tan x}{\mathrm{d}x}=\sec^2 x \qquad\qquad \frac{\mathrm{d}\cot x}{\mathrm{d}x}=-\csc^2 x$$

$$\frac{\mathrm{d}\sec x}{\mathrm{d}x}=\sec x\tan x \qquad\qquad \frac{\mathrm{d}\csc x}{\mathrm{d}x}=-\csc x\cot x$$

Integration: the inverse process of differentiation

$$\int\cos x\,\mathrm{d}x=\sin x+C \qquad\qquad \int\sin x\,\mathrm{d}x=-\cos x+C$$

$$\int\sec^2 x\,\mathrm{d}x=\tan x+C \qquad\qquad \int\csc^2 x\,\mathrm{d}x=-\cot x+C$$

$$\int\sec x\tan x\,\mathrm{d}x=\sec x+C \qquad\qquad \int\csc x\cot x\,\mathrm{d}x=-\csc x+C$$