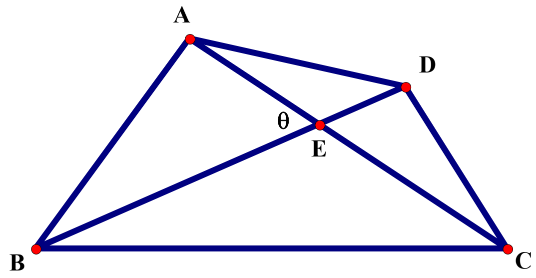
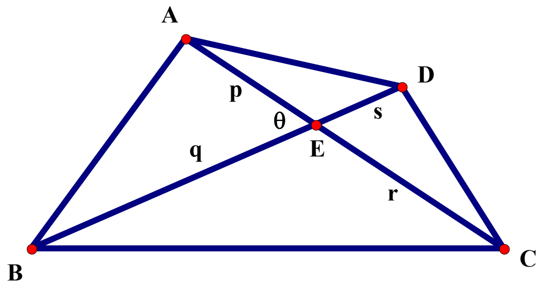
# Area of quadrilateral

Created by Mr. Francis Hung Last updated: February 16, 2024

In a quadrilateral *ABCD*. Given $AC = x$, $BD = y$ and $\angle AEB = \theta$, find the area of a quadrilateral.




Suppose the diagonals *AC* and *BD* intersect at *E*.

Let $AE = p$, $BE = q$, $CE = r$, $DE = s$.

Then $\angle CEB = 180° - \theta$ (adj. ∠s on st. line)

$\angle CED = \theta$ (vert. opp. ∠s)

$\angle AED = 180° - \theta$ (adj. ∠s on st. line)

Area of *ABCD* = area of Δ*ABE* + area of Δ*BCE* + area of Δ*CDE* + area of Δ*ADE*

$$= \frac{1}{2}pq\sin\theta + \frac{1}{2}qr\sin(180° - \theta) + \frac{1}{2}rs\sin\theta + \frac{1}{2}ps\sin(180° - \theta)$$

$$= \frac{1}{2}pq\sin\theta + \frac{1}{2}qr\sin\theta + \frac{1}{2}rs\sin\theta + \frac{1}{2}ps\sin\theta$$

$$= \frac{1}{2}\sin\theta(pq + qr + rs + ps)$$

$$= \frac{1}{2}\sin\theta[p(q+s) + r(q+s)]$$

$$= \frac{1}{2}\sin\theta(p+r)(q+s)$$

$$= \frac{1}{2}xy\sin\theta$$

**Example 1**: If $AC = x = 8$, $BD = y = 6$ and $\angle AEB = \theta = 60°$

Area of *ABCD* = $\frac{1}{2}\cdot 8\cdot 6\sin 60° = 24\times\frac{\sqrt{3}}{2} = 12\sqrt{3}$

**Example 2** If $AC = 10$, $BD = 8$ and $AC \perp BD$

Area of *ABCD* = $\frac{1}{2}\cdot 8\cdot 10\sin 90° = 40$