To evaluate $\lim_{x\to\infty}\frac{\sin x}{x}$.

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Squeezing principle (Sandwich principle) for limits

Suppose $g(x) \le \mathrm{f}(x) \le h(x)$ for all x such that $a \le x \le b$

and also suppose that $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = M$, where $a \le c \le b$.

Then $\lim_{x\to c}\mathrm{f}(x)$ exists and equal to M.

Evaluate $\lim_{x\to\infty}\frac{\sin x}{x}$.

$-1 \le \sin x \le 1$, where x is in radians.

$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$, where $x > 0$.

$\lim_{x\to\infty}\left(-\frac{1}{x}\right) = \lim_{x\to\infty}\left(\frac{1}{x}\right) = 0$

Then by squeezing principle, $\lim_{x\to\infty}\frac{\sin x}{x}$ exists and equal to 0.

Similarly, $-\frac{1}{x} \ge \frac{\sin x}{x} \ge \frac{1}{x}$, where $x < 0$.

$\lim_{x\to\infty}\left(-\frac{1}{x}\right) = \lim_{x\to\infty}\left(\frac{1}{x}\right) = 0$

Then by squeezing principle, $\lim_{x\to-\infty}\frac{\sin x}{x}$ exists and equal to 0.