Differentiation of Implicit function Example
Created by Mr.Francis Hung on 20220212. Last updated: 12 February 2022.

M2 Exercise 8E Q15

Method 1

$$y = \frac{x-y}{x+y}$$

$$xy + y^2 = x - y \quad \cdots\cdots (*)$$

$$x = \frac{y^2 + y}{1-y} \quad \cdots\cdots (**)$$

Differentiate (*) w.r.t. x.

$$\frac{d}{dx}\left(xy + y^2\right) = \frac{d}{dx}(x-y)$$

$$x\frac{dy}{dx} + y + 2y\cdot\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1-y}{x+2y+1} \quad \cdots\cdots (1)$$

Sub. (**) into (1)

$$\frac{dy}{dx} = \frac{1-y}{\dfrac{y^2+y}{1-y}+2y+1} = \frac{(1-y)^2}{y^2+y+(2y+1)(1-y)} = \frac{(1-y)^2}{y^2+y-2y^2+y+1} = \frac{(1-y)^2}{-y^2+2y+1} \quad \cdots\cdots (2)$$

Method 2

$$y = \frac{x-y}{x+y}$$

$$\frac{dy}{dx} = \frac{(x+y)\left(1-\dfrac{dy}{dx}\right) - (x-y)\left(1+\dfrac{dy}{dx}\right)}{(x+y)^2}$$

$$(x+y)^2\frac{dy}{dx} = (x+y)\left(1-\frac{dy}{dx}\right) - (x-y)\left(1+\frac{dy}{dx}\right)$$

$$(x+y)^2\frac{dy}{dx} = x+y - (x+y)\frac{dy}{dx} - (x-y) - (x-y)\frac{dy}{dx}$$

$$[(x+y)^2 + (x+y) + (x-y)]\frac{dy}{dx} = 2y$$

$$\frac{dy}{dx} = \frac{2y}{(x+y)^2 + 2x} \quad \cdots\cdots (3)$$

Sub. (**) into (3):

$$\frac{dy}{dx} = \frac{2y}{\left(\dfrac{y^2+y}{1-y}+y\right)^2 + 2\cdot\dfrac{y^2+y}{1-y}} = \frac{2y(1-y)^2}{\left(y^2+y+y-y^2\right)^2 + 2\left(y^2+y\right)(1-y)} = \frac{2y(1-y)^2}{(2y)^2 + 2\left(y^2+y-y^3-y^2\right)}$$

$$= \frac{y(1-y)^2}{2y^2+y-y^3} = \frac{(1-y)^2}{-y^2+2y+1} \quad \cdots\cdots (4)$$

The results are the same.