<u>Limit Integral to find the area under the curve $y = \cos x$, $0 \le x \le 0.5\pi$</u>

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$$\int_0^{\frac{\pi}{2}} \cos x\,dx = \lim_{n\to\infty}\left(A_1 + A_2 + \cdots + A_n\right)$$

$$= \lim_{n\to\infty}\left(\frac{\pi}{2n}\cos\frac{\pi}{2n} + \frac{\pi}{2n}\cos\frac{2\pi}{2n} + \cdots + \frac{\pi}{2n}\cos\frac{n\pi}{2n}\right)$$

$$= \lim_{n\to\infty}\frac{\pi}{2n}\left(\cos\frac{\pi}{2n} + \cos\frac{2\pi}{2n} + \cdots + \cos\frac{n\pi}{2n}\right)$$

$$= \lim_{n\to\infty}\frac{\pi}{4n\sin\frac{\pi}{4n}}\left(2\cos\frac{\pi}{2n}\sin\frac{\pi}{4n} + 2\cos\frac{2\pi}{2n}\sin\frac{\pi}{4n} + \cdots + 2\cos\frac{n\pi}{2n}\sin\frac{\pi}{4n}\right)$$

$$= \lim_{n\to\infty}\frac{\pi}{4n\sin\frac{\pi}{4n}}\left[\sin\frac{3\pi}{4n} - \sin\frac{\pi}{4n} + \sin\frac{5\pi}{4n} - \sin\frac{3\pi}{4n} + \cdots + \sin\frac{(2n+1)\pi}{4n} - \sin\frac{(2n-1)\pi}{4n}\right]$$

$$= \lim_{n\to\infty}\frac{\pi}{4n\sin\frac{\pi}{4n}}\left[\sin\frac{(2n+1)\pi}{4n} - \sin\frac{\pi}{4n}\right]$$

$$= \lim_{n\to\infty}\frac{\pi}{4n\sin\frac{\pi}{4n}}\left[2\cos\frac{(n+1)\pi}{4n}\sin\frac{n\pi}{4n}\right]$$

$$= \frac{1}{\lim\limits_{n\to\infty}\frac{\sin\frac{\pi}{4n}}{\frac{\pi}{4n}}}\cdot 2\cdot\lim_{n\to\infty}\cos\left[\frac{\pi}{4}\cdot\left(1+\frac{1}{n}\right)\right]\sin\frac{\pi}{4}$$

$$= 1\cdot 2\cdot\cos\frac{\pi}{4}\cdot\sin\frac{\pi}{4}$$

$$= \sin\frac{\pi}{2} = 1$$