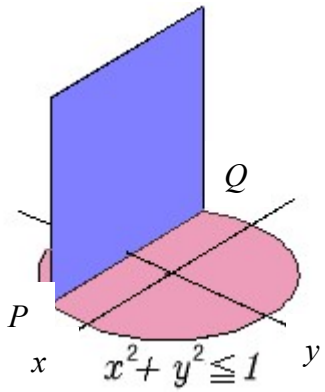
Volume

Created by Mr. Francis Hung on 20220422. Last updated: 26 February 2023

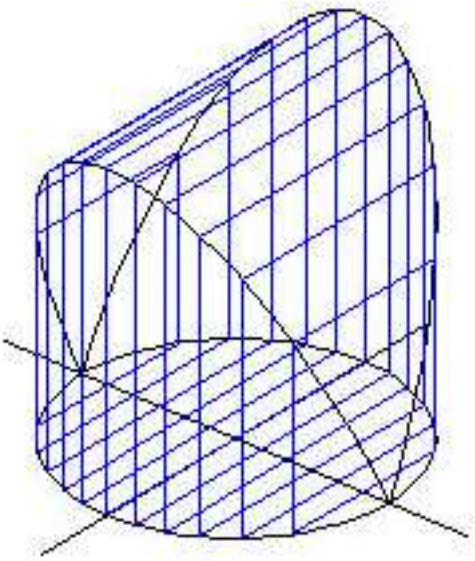
There is a solid whose bottom face is the circle $x^2 + y^2 \le 1$ and every cross-section of the solid perpendicular to x-axis is a square. Find the volume of the solid.



Let P, Q be the 2 feet of the square.

$P = (\sqrt{1-y^2}\ , y)$, $Q\ (-\sqrt{1-y^2}\ , y)$

$PQ = 2\sqrt{1-y^2}$



Area of a slice of square = $4(1 - y^2)$

$dV = 4(1 - y^2)\ dy$

Volume V = $\int_{-1}^{1} 4\left(1-y^2\right) dy$

$= 8\int_{0}^{1}\left(1-y^2\right) dy$

$= 8\left[y - \frac{y^3}{3} \right]_0^1$

$= 8 \times \frac{2}{3} = \frac{16}{3}$