Browver's Theorem (Fixed Point Theorem)

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Let $f:[a, b] \to [a, b]$ be continuous, then $\exists x_0 \in [a, b]$ s.t. $f(x_0) = x_0$

Proof: Consider $g(x) = f(x) - x$, $x \in [a, b]$, then g is continuous

$g(a) = f(a) - a \geq a - a = 0$

$g(b) = f(b) - b \leq b - b = 0$

$\therefore g(a)g(b) \leq 0$

By intermediate value theorem, $\exists x_0 \in [a, b]$ such that $g(x_0) = 0$

i.e. $f(x_0) = x_0$