2 circles touch each, find the point of contact

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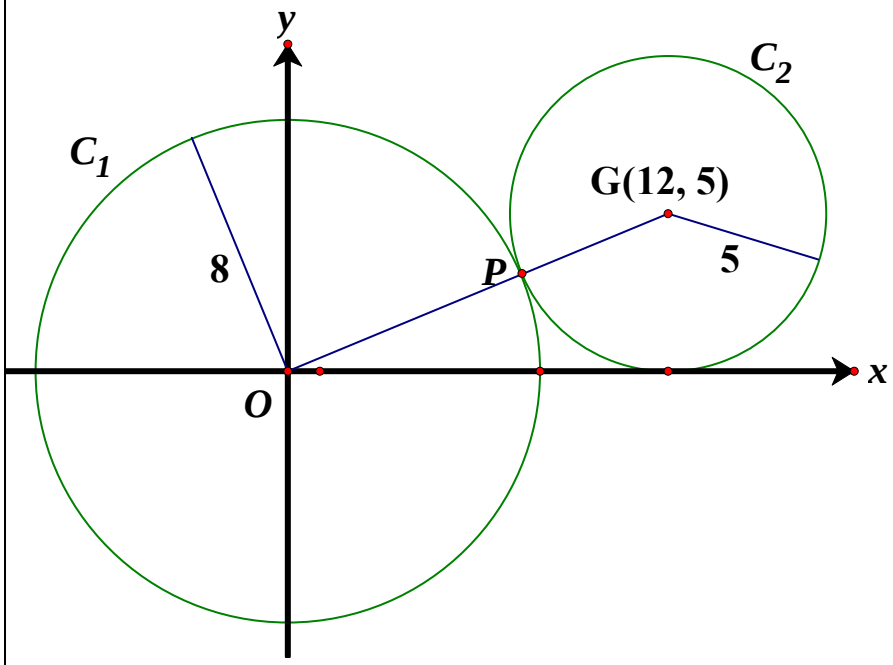


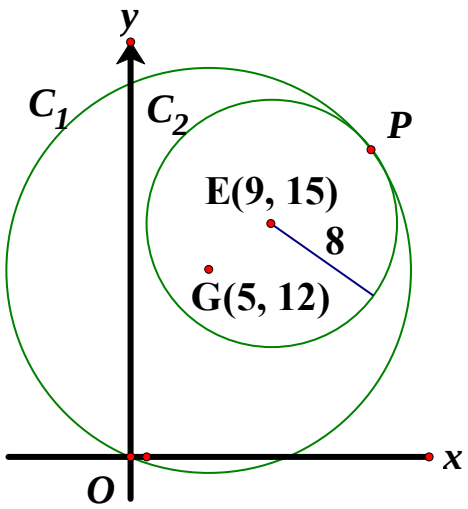
Given two circles C_1, centre at origin $O(0, 0)$, radius 8 and C_2, centre at $G(12, 5)$, radius 5. If C_1 and C_2 touch each other externally at P, find the coordinates of P.

Distance between centres = $OG = \sqrt{(12-0)^2+(5-0)^2} = 13$

Sum of radii = 8 + 5 = 13 = $OP + GP = OG$

$\therefore$ O, P, G are collinear and $OP : PG = 8 : 5$

By section formula, $P = \left(\frac{5\times 0+8\times 12}{8+5}, \frac{5\times 0+8\times 5}{8+5}\right) = \left(\frac{96}{13}, \frac{40}{13}\right)$.



Given two circles C_1, centre at origin $G(5, 12)$, passing through the origin O and C_2, centre at $E(9, 15)$, radius 8. If C_1 and C_2 touch each other internally at P, find the coordinates of P.

Radius of $C_1 = \sqrt{(12-0)^2+(5-0)^2} = 13$

Distance between centres = $EG = \sqrt{(9-5)^2+(15-12)^2} = 5$

Difference of radii = 13 – 8 = 5 = $GP - EP = GE$

$\therefore$ P, E, G are collinear and $GE : EP = 5 : 8$

Let the coordinates of P be (x, y).

By section formula, $\left(\frac{8\times 5+5x}{8+5}, \frac{8\times 12+5y}{8+5}\right) = E(9, 15)$

$5x + 40 = 9\times 13$, $x = \frac{77}{5}$; $5y + 96 = 15\times 13$, $y = \frac{99}{5}$. $P = \left(\frac{77}{5}, \frac{99}{5}\right)$.