# Family of circles Example

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**Example** Find the equation of circle which passes through $A(1, 2)$, $B(5, 4)$ and touches $4y = 3x$.

Equation of $AB$: $\dfrac{y-2}{x-1}=\dfrac{4-2}{5-1}$

$2y - 4 = x - 1$

$x - 2y + 3 = 0$ (call it $L$)

Equation of circle with $AB$ as diameter: $(x - 1)(x - 5) + (y - 2)(y - 4) = 0$

$x^2 + y^2 - 6x - 6y + 13 = 0$ (call it $C$)

Family of circles pass through $AB$: $C + kL$: $x^2 + y^2 - 6x - 6y + 13 + k(x - 2y + 3) = 0$

$x^2 + y^2 + (k - 6)x - (6 + 2k)y + 13 + 3k = 0$

centre = $G = \left(\dfrac{6-k}{2}, k+3\right)$

radius = $r = \sqrt{\left(\dfrac{6-k}{2}\right)^2+(k+3)^2-(13+3k)}$

$$= \frac{1}{2}\sqrt{36-12k+k^2+4(k^2+6k+9)-52-12k}$$

$$= \frac{1}{2}\sqrt{5k^2+20}$$

$\because$ It touches $4y = 3x$

$\therefore$ Distance from centre $G$ to $3x - 4y = 0$ is the radius

$$\left|\frac{3\times\frac{6-k}{2}-4\times(k+3)}{\sqrt{3^2+4^2}}\right| = \frac{1}{2}\sqrt{5k^2+20}$$

$$\left|-6-11k\right| = 5\sqrt{5k^2+20}$$

$(6 + 11k)^2 = 25(5k^2 + 20)$

$36 + 132k + 121k^2 = 125k^2 + 500$

$4k^2 - 132k + 464 = 0$

$k^2 - 33k + 116 = 0$

$(k - 4)(k - 29) = 0$

$k = 4$ or $k = 29$

When $k = 4$, $H_1$: $x^2 + y^2 - 2x - 14y + 25 = 0$

When $k = 29$, $H_2$: $x^2 + y^2 + 23x - 64y + 100 = 0$